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052 Improvement of Iceberg Drift Forecast — Grand Banks

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AN OPERATIONAL ICEBERG TRAJECTORY FORECASTING MODEL FOR THE GRAND BANKS OF NEWFOUNDLAND

Sylvain de Margerie[1]
John Middleton[2]
Chris Garrett[2]
Steven Marquis[1]
Fouad Majaess[2]
Karen Lank[1]

[1] ASA Consulting Ltd.
P.O. Box 2025, Dartmouth East
Dartmouth, Nova Scotia, B2W 3X8

[2] Dalhousie University
Halifax, Nova Scotia, B3H 4J1

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SUMMARY

Several operational forecast models are now available for iceberg trajectory prediction. Most, however, are based on an exact solution of the force balance equation for the iceberg and, therefore, assume implicitly that these forces are quantifiable in a forecast situation. In reality this is not the case as many factors, such as iceberg geometry and future currents or winds, are at best imperfectly known and degrade the accuracy of prediction. The principal shortcoming of deterministic models, however, stems from our inability to forecast oceanic eddies which account for a large part of iceberg motion.

Here we take a different approach towards iceberg forecasting, treating the effects of uncertainties in environmental parameters and ocean turbulence in a statistical manner. We reason that even if we cannot predict these deterministically, valuable information may be drawn from past iceberg observations allowing us to predict a most likely future position. Although the major part of this model is based on statistical techniques, some components, such as tidal motion, are accurately predictable and are treated deterministically. A basic requirement for the implementation of this model is an extensive analysis of past iceberg behaviour in the region of concern.

For the Grand Banks of Newfoundland, the 1984 and 1985 iceberg observations, taken from oil rigs as well as from their support vessels and aircraft, were available for our analysis. After elimination of dubious data, the first step of the analysis consisted of determining the iceberg response to wind. A correlation analysis showed that icebergs exhibit a downwind drift of 1.8% of the 10 m wind speed, confirming results of observations on the Labrador Shelf. Next, the wind-driven as well as the tidal components of motion were subtracted from the observed iceberg velocities to obtain the residual drift.

The analysis of residual iceberg velocities on the Grand Banks allows us to determine the mean pattern as well as the amount of variability in iceberg motion over the study area. Icebergs clearly show the effect of the Labrador Current, which flows along the edge of the Grand Banks, causing a strong mean drift and increased variance.

An autocorrelation analysis of the variable part of the residual iceberg motion shows that the drift tends to be consistent over times in the order of 25 hours. This means that in a forecast situation one may be able to infer effectively the future velocity of an iceberg simply by observing its present rate of drift (and appropriately accounting for wind and tidal motion). Such a prediction is relatively accurate for times shorter than 25 hours; however, as time progresses, the velocity gradually loses correlation with its initial value and becomes less predictable.

Based on the results of these analyses an operational forecast model is conceived. Model inputs consist of observed and forecast winds as well as the most recent iceberg positions. Model parameters include the iceberg decorrelation time scale (25 hours), wind drift factor (1.8%), tidal constants, mean residual velocities, and variances, all of which are defined

on a grid system to reproduce their spatial variabilities. A standard technique is used to compute tidal current; 1.8% of the wind speed and the mean flow are combined to obtain the deterministic component of drift velocity. This velocity is subtracted from the velocity estimated from recent iceberg position fixes to assess the residual iceberg velocity, which is the component of motion that tends to persist for time scales of about 25 hours. This motion is treated as a fading memory process with the velocity decreasing exponentially from its initial value, with a time constant of 25 hours. To compute future positions, the effects of the velocity perturbations are recombined with predicted tidal currents, the mean flow and the future wind drift (using the weather forecast). Position calculations are done at hourly intervals. For each predicted position, confidence intervals are computed and the chance of iceberg impact with the rig is assessed. By comparing the probability of impact to allowable risk the model can provide guidance for the initiation of evasive action.

The actual operational model is implemented to run on any IBM-compatible microcomputer. The operation of the model and the interpretation of the results are made easy through extensive use of the graphic and interactive capabilities of microcomputers. In addition to providing a forecasting tool the model automatically archives observed iceberg tracks, thus reducing the burden on the iceberg observers.

RESUME

Plusieurs modèles numériques sont présentement disponibles pour la prévision de trajectoires d'icebergs. La plupart d'entre eux se base sur la solution exacte de l'équation des forces agissant sur l'iceberg, prenant ainsi pour acquis que ces forces sont quantifiables. En fait ces forces dépendent de plusieurs facteurs, tel que la géometrie de l'iceberg ou les vitesses du vent et du courant, cependant ces deux derniers facteurs sont au mieux, mal connus et par conséquent rendent les prévisions peu fiables. A l'heure actuelle, la lacune principale de ces modèles déterministes provient toutefois de l'inabilité à prévoir le mouvement aléatoire des gires océaniques qui contribuent substantiellement au mouvement des glaces.

Nous adoptons ici une approche différente, en appliquant un traitement statistique à la composante de mouvement attribuable aux gires océaniques ou à d'autres effets impondérables. Bien que ceux-ci ne soient pas prévisibles d'une façon déterministe, un examen des observations passées pourrait quand même nous fournir une information utile pour déterminer la position probable des icebergs dans le futur. Notre modèle se base principalement sur un traitement statistique mais inclus tout de même certaines composantes déterministes qui sont prévisibles tel que le mouvement dû à la marée. Il est essentiel pour la mise en place de ce modèle de procéder à une analyse complète des données existantes sur les icebergs, afin de caractériser les propriétés statistiques de leur dérive pour la région concernée.

Les données des années 1984 et 1985 ont été analysées pour les Grands Bancs. Après l'élimination de toutes levées douteuses, la première tâche consiste à déterminer l'effet du vent sur la dérive des icebergs. Une analyse de corrélation nous permet d'établir qu'en moyenne les icebergs dérivent et dans la direction du vent à 1.8% de la vitesse de celui-ci confirmant ainsi les résultats de Garrett et al pour le plateau du Labrador. Pour la prochaine étape d'analyse, le mouvement attribuable au vent et à la marée sont soustraits des trajectoires observées afin d'obtenir la composante de dérive résiduelle.

Une analyse de cette dérive résiduelle nous permet de quantifier le mouvement moyen et la variance des trajectoires des icebergs. La distribution bi-dimensionnelle de ces quantités montre clairement que le courrant du Labrador longe le rebord du plateau continental. En plus d'un accroissement de la dérive moyenne, celui-ci semble engendrer un augmentation de la variance.

Une analyse d'autocorrélation des vitesses résiduelles des icebergs nous permet d'établir que la dérive démontre une certaine constance sur une échelle de temps inférieure à 25 heures. Il est ainsi possible de baser une prévision sur la vitesse présente ou passée d'un iceberg (en tenant compte aussi des effets du vent et de la marée). Une telle prévision peut être assez précise à court terme, mais devient moins fiable au delà de 25 heures.

C'est en prenant comme point de départ les résultats de nos analyses qu'un modèle de prévision de trajectoire d'iceberg a été conçu. Les données d'entrée se composent d'observations et de prédiction du vent ainsi que de levées récentes de la position sur l'iceberg en question. Les paramètres du modèle comprennent l'échelle de temps de corrélation (25 heures), le facteur de dérive due au vent (1.8%), ainsi que les constantes de marée, la dérive moyenne et la variance résiduelle qui sont définies sur une grille bi-dimensionnelle. Les courants de marée, calculés selon la méthode habituelle, la dérive moyenne et la dérive due au vent sont soustraits de la vitesse observée résultante de l'iceberg (calculée à partir de levées récentes) afin d'obtenir la vitesse résiduelle. Celle ci est en fait la partie du mouvement qui montre une constance sur une échelle temporelle de 25 heures. Cette caractéristique du mouvement est modélisée par un processus à mémoire décroissante; la vitesse décroit exponentiellement à partir de sa valeur initiale sur un échelle de 25 heures. Afin d'arriver à une prédiction, cette vitesse décroissante s'ajoute au déplacement moyen, à la dérive prédite pour la marée et à la dérive éoléenne (basée sur les prédictions météorologiques). Les positions futures de l'iceberg sont calculées à intervalle d'une heure. En plus de la position des icebergs, les limites de fiabilité et la probabilité d'impact contre la platte-forme de forage sont estimées. Cette dernière, lorsque comparée au risque permissible, peut servir de guide décisionnelle pour initier les procédures d'évacuation.

Le modèle est installé sur un micro-ordinateur du type IBM qui sied bien à l'usage sur le terrain. La pleine capacité du micro-ordinateur est utilisée pour simplifier l'opération du modèle. Les résultats sont présentés de façon claire pour l'utilisation d'une méthode graphique et interactive.

INTRODUCTION

Icebergs are a major concern for oil exploration in eastern Canadian waters. Offshore exploration platforms are not designed to withstand the impact of icebergs, and although small icebergs may be towed away, the only effective countermeasure is often evasive action. In view of the significant cost of interrupting drilling operations by disconnecting and then re-entering the well, some method is required to determine whether or not a particular iceberg represents a significant threat to the platform. Several numerical models have been used to predict iceberg trajectories in attempts to provide such information. Many of them use deterministic prediction schemes and have fallen in disfavour due to their lack of accuracy. The reason for this apparent failure can be found in the turbulent nature of ocean circulation, where eddy-like motion contributes significantly to iceberg behaviour. This motion cannot at present be predicted deterministically, giving rise to the common remark that icebergs must have minds of their own (LeBlond and Hodgins, 1985).

In a recent study of iceberg motion on the Labrador Shelf, Garrett et al. (1985) suggested an alternative approach to forecasting iceberg trajectories. This approach considers iceberg motion principally from a statistical point of view. From a careful examination of iceberg behaviour it is found that a certain part of the eddy motion can be predicted by considering the past history of a particular iceberg. This technique, combined with a simple consideration of tides, wind, and mean drift, has the potential of significantly improving iceberg forecasts, but remains computationally simple. The technique also offers a method of computing the confidence intervals which indicate to the user the expected level of accuracy for each prediction.

Inspired by the work of Garrett et al. (1985), ASA Consulting Ltd. proposed to implement this technique into an on-site operational iceberg forecast model for the Grand Banks of Newfoundland. The Environmental Studies Revolving Funds (ESRF) is funding the present project. The study area (Figure 1) covers the region of most intense oil exploration on the Grand Banks, where icebergs represent an operational concern.

The implementation of the operational model for the Grand Banks necessitates several modifications to the Garrett et al. (1985) conceptual model because of site-specific environmental conditions and operational procedures. To arrive at a practical forecasting system, one must also carefully consider ease of use and interpretation. This report documents in detail all aspects of the operational model. In its present state, the forecasting program has been verified against observations, and should provide a useful tool for predicting iceberg trajectories on the Grand Banks. Further improvements are, however, undoubtedly possible, as a result of comments during the field trials and as more iceberg observations become available for analysis.

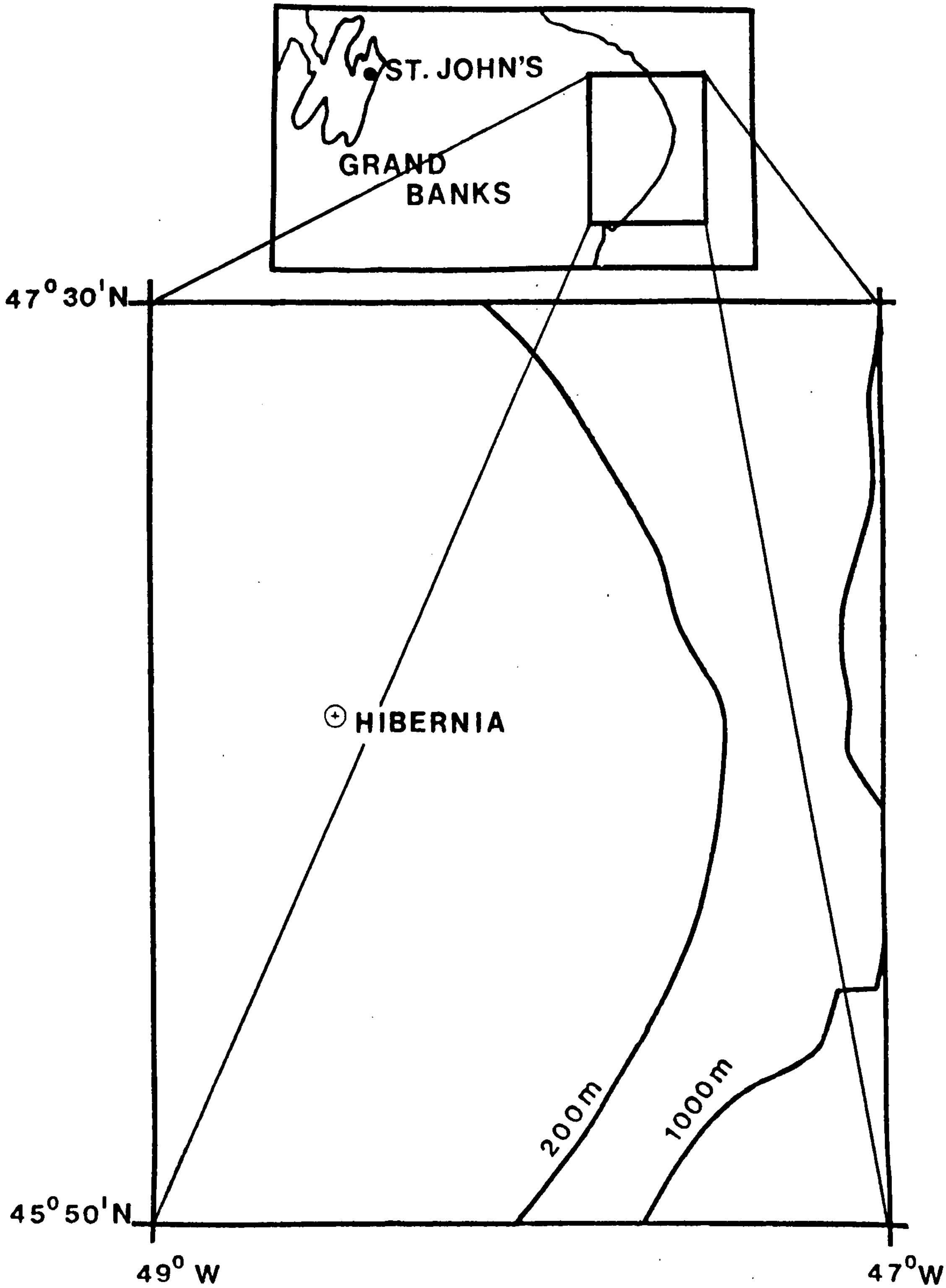

Figure 1. Study area on Grand Banks of Newfoundland.

CHAPTER 1 GENERAL MODEL THEORY

Iceberg trajectory forecast models can be grouped in three general categories: deterministic, statistical, and hybrid. Operational forecast models have traditionally used the deterministic approach, even though recent research has shown promise in statistical and hybrid models (Whitemore and Gentleman, 1985; Gaskill et al., 1984). Before discussing the theoretical grounding of our operational hybrid model, let us first examine the three types of model in more detail (a more thorough review can be found in Marko et al., 1983).

Deterministic models are based on exact solutions of the equations of motion for icebergs (Smith and Banke, 1983; El Tahan, 1980). To achieve this one must be able to evaluate all forces acting on an iceberg. Unfortunately these forces depend on iceberg geometry, wave climate, wind and current velocities, all of which are at best imperfectly known in an operational forecast situation. The resulting models often show encouraging results in hindcast studies when carefully measured iceberg shapes and observed atmospheric and ocean climate data are available to calculate the forces at each point along the iceberg's trajectory. In an operational situation, however, these models suffer from prediction errors resulting from uncertainties in all of these quantities. Deterministic models often do not provide explicit estimates of prediction error.

In statistical models, data analysis techniques are used to determine the statistical behaviour of observed iceberg tracks (Whitemore and Gentleman, 1985; Intera, 1980). If one has enough observations and can assume that iceberg behaviour will not change systematically in the future, these results can be used to predict probable iceberg positions so as to minimize mean squared error. The analysis may include fitting relationships between future iceberg positions and known quantities such as past iceberg position, observed or predicted winds. The model predictions always retain a certain amount of quantifiable uncertainty which may be the result of random effects of unpredictable atmospheric and oceanic fluctuations.

Hybrid models borrow from both deterministic and statistical models in varying proportions (Gaskill et al., 1984; Garrett, 1984; Garrett et al., 1985). They assume that part of the iceberg motion can be predicted exactly but there always remains a certain portion of motion which can be regarded as resulting from a random process. These models may show significant improvements over purely statistical models if a large fraction of the motion results from predictable quantities such as tidal currents. Some hybrid models provide information on the confidence level of predicted positions.

Many of the iceberg trajectory prediction models now available can be considered of a hybrid nature. However, most remain largely based on the deterministic philosophy and give little consideration to the predictive value of complex deterministic formulations. Here we put forth a complementary approach, starting from a purely probabilistic approach and adding deterministic components only if it can be shown that they significantly improve predictive ability. This approach results in a simple and computationally efficient model. In the present implementation only tides are considered in a deterministic fashion. Forecast winds are used in the prediction, but the coupling between wind and iceberg motion is determined from statistical analysis. Similarly the mean drift is obtained from statistical analysis. It can be expected that as reliable current forecast models and observational techniques become available their predictions could be incorporated and used to some advantage in the present model.

1.1 OPERATIONAL MODEL

The operational model to be implemented here for the Grand Banks is based on the work of Garrett et al. (1985) who investigated the behaviour of icebergs on the Labrador Shelf. Their research has led them to propose a prediction scheme for iceberg trajectories based on statistical analysis but which can also include deterministic components. The particular implementation proposed here can be formulated as:

$$z(T) = z(0) + \int_0^T \left[U_t(t) + \alpha_w W(t) + \overline{U} \right] dt + \alpha(T)\, U(0) + \epsilon(T)$$

where z is the iceberg position,

T is the time of the prediction, measured from the origin, (T=0), which corresponds to the most recent iceberg observation,

U_t is the tidal velocity,

W is the wind speed (preferably near-surface wind, i.e., 10 m),

α_w is a wind drift coefficient,

$\overline{U}$ is the climatological mean iceberg drift velocity,

α is a time-dependent prediction factor (not related to α_w),

U(0) is a recently observed residual velocity, and

ϵ is the prediction error.

The second right-hand term groups the effects of tides, winds, and mean drift. The tidal currents are considered deterministic and are applied directly to the iceberg. The winds are also considered as deterministic (i.e., the error in wind forecast is not considered) but the response of the iceberg to wind (α_w) is determined from a statistical analysis. The mean iceberg drift is similarly determined from an analysis of observed iceberg tracks and is considered deterministically.

The third right-hand term is a statistical prediction term using previously observed iceberg velocity estimates as predictors. The statistical component represents mostly the effect of oceanic eddies which at present cannot be forecast deterministically. Although the conceptual model can use several past velocities in the prediction (or even other predictors independent of iceberg observations), it was shown by Garrett et al. (1985) that little advantage is generally gained by using many predictors and it suffices to consider a single predictor consisting of a recent velocity observation. The iceberg velocity, U(0), used here is really a velocity perturbation, as the effects of tides, wind, and mean drift have been removed. The coefficient α is a function of time determined from an autocorrelation analysis of iceberg observations, and in the simplest case is given by:

$$\alpha = \gamma \ (1-e^{-\gamma t})$$

where γ^{-1} is a time scale (specifically the Lagrangian time scale), which expresses the fact that an iceberg is likely to keep its present course for a period in the order of γ^{-1}.

The last term is the prediction error which also depends on the iceberg behaviour as determined from autocorrelation analysis. This term allows a determination of confidence intervals in predicted positions and the evaluation of impact probability.

1.2 TIDE-INDUCED MOTION OF ICEBERGS

Tidal currents in the first few tens of metres of the water column can contribute significantly to iceberg motion. Inasmuch as these tidal currents are predictable, they will contribute a deterministic component to iceberg motion. A simple dynamical analysis can show that the icebergs should respond directly to ambient currents (i.e., the tidal current velocity is entirely imparted to the iceberg). In general, current shear may exist on the scale of iceberg dimensions, especially in the vertical, complicating the relationship between iceberg and water motion. In predicting the motion of icebergs it would therefore be preferable to determine the tidal constants from observed iceberg trajectories.

Tides are a direct result of the gravitational attraction of the sun and moon. Because these forces are cyclical and can be predicted accurately, the responses they cause in the ocean are largely deterministic (although secondary effects such as variability in stratification, if one considers internal tides, may induce non deterministic components). For the purposes of analysis and prediction the tidal signal is generally decomposed into a series of constituents of constant frequency. The amplitude and phase lag of each of these constituents is determined from the analysis of observations, and velocity predictions for each component (x and y) are given by:

$$u_t = \sum_i u_i \cos(\omega_i t - P_i) \qquad (1.1)$$

where u_i, ω_i, and P_i are the amplitude, frequency, and phase of the i^{th} constituent.

The results of Garrett et al. (1985) show good agreement between tidal currents deduced from iceberg motion and those observed directly with current meters in the upper water column. To consider the statistical behaviour of the residual eddy motion, the tidal motion, which can be considered deterministically, is removed from the observed tracks.

1.3 WIND-INDUCED MOTION OF ICEBERGS

The tideless iceberg velocity field can be modelled by (Garrett et al., 1985):

$$U_b = U_o + \alpha_w W \qquad (1.2)$$

where U_o denotes a near-surface water velocity, α_w is a scalar drift factor, and W is the wind velocity. In this formula the iceberg response is assumed to be linear and instantaneous, which can be shown to be an appropriate approximation from a dynamical point of view (Garrett et al., 1985). Cross-correlation analysis of iceberg and wind data for the Labrador Shelf shows that icebergs drift at 1.8% of the wind speed, with negligible angular deflection. This drift coefficient is consistent with those derived from an analysis of drag forces on icebergs. It should be noted that this drift includes the indirect effects of winds, such as wind-induced currents and waves, therefore decreasing the advantage to be gained from a deterministic inclusion of these factors. Before considering the behaviour of residual iceberg velocities, the component of wind-induced motion is removed, as it was for the tide. In the prediction model, tidal, wind, and mean motion are added to the statistical prediction of eddy motion. In doing this we are neglecting errors resulting from wind forecast uncertainties. Considering that the prediction uncertainty resulting from the randomness of eddy motion is usually large, compared to the wind driven motion itself, the added error introduced by the use of forecasted wind is small.

1.4 MEAN RESIDUAL ICEBERG VELOCITY

After removal of the wind- and tide-induced motion, observed iceberg tracks can be analysed for any residual mean motion. This mean component results from the mean oceanic circulation and could be obtained from other sources such as current meter records or drifter data; however, as for tides, the mean drift is best determined from the icebergs themselves. We should note that the mean flow discussed here does not include an average component of drift resulting from wind, which has already been removed (see Section 1.3). The mean residual component of iceberg drift is considered deterministically in the prediction model.

1.5 STATISTICAL MODEL

The following discussion is based, with few exceptions, on Chapter 7 of Garrett et al. (1985) and on Garrett (1985), to both of which the reader is directed for a more detailed and rigorous discussion. The general problem of statistical forecasting consists of determining the best estimate, $\hat{x}$, of a quantity x, from a linear combination of inputs or predictors, y_i, according to the formula:

$$\hat{x} = \Sigma \alpha_i' y_i \tag{1.3}$$

Here we define the best estimate as the one which minimizes the mean square error, $\overline{(x - \hat{x})^2}$. Given a set of observed x and corresponding inputs y_i, one can determine the coefficients α_i' that are required for optimum prediction.

Let us now consider more specifically the motion of icebergs with any mean and any deterministic component removed and with no cross-correlation between the u and v components (equivalent to a one-dimensional case). Future iceberg velocities U'(t) are to be predicted from past iceberg velocities $U'(-t_i)$. The coefficients α_i' can be related to the autocorrelation function of iceberg velocities, $R(\tau) = \overline{U'(t)\, U'(t+\tau)} / \overline{U'(t)\, U'(t)}$, as a function of lag, τ, by:

$$\begin{bmatrix} 1 & R(t_2-t_1) & R(t_3-t_1) & \cdots \\ R(t_1-t_2) & 1 & R(t_3-t_2) & \cdots \\ \vdots & & \ddots & \end{bmatrix} \begin{bmatrix} \alpha'_1 \\ \alpha'_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} R(t+t_1) \\ R(t+t_2) \\ \vdots \end{bmatrix} \qquad (1.4)$$

In the case where $R(\tau) = e^{-\gamma \tau}$, it can be shown that the optimum estimate is obtained by using only the most recent velocity observation and a coefficient $\alpha' = e^{-\gamma t}$. An exponential autocorrelation function can be a good first approximation to observed iceberg autocorrelation functions and arises from familiar random walk or first-order Markov processes. For such processes future velocities are simply related to past velocities by:

$$U'(t) = e^{-\gamma t} U'(0) + \epsilon(t) \qquad (1.5)$$

where ϵ is a normally distributed random variable, uncorrelated with u, with zero mean and variance equal to $\overline{(U')^2}(1-e^{-2\gamma t})$. In equation 1.5, the first term on the right-hand side, $e^{-\gamma t} U'(0)$, is the optimum estimate of $U'(t)$, whereas the second term gives us an estimate of the possible prediction error.

The exponential autocorrelation function expresses the fact that an iceberg is likely to retain its present velocity for some time in the future. With time, however, successive velocity perturbations will progressively obliterate all correlation with an initial velocity. The iceberg effectively has a fading memory of its previous velocity. The quantity γ describes the rate at which the iceberg velocity loses its correlation with an initial value, and is appropriately called the decorrelation rate. In our case the velocity estimates move with the iceberg (i.e., successive estimates are for different points along the iceberg trajectory), and γ^{-1} is referred to as the

Lagrangian decorrelation time scale. It can be noted that winds and currents usually exhibit a decorrelation in time so that the simple model discussed represents the combined effect of wind and currents on iceberg motion.

It should be noted that apart from wind effects, which have been removed here, icebergs follow the movement of the ambient water. The statistical behaviour of icebergs therefore directly reflects the properties of the oceanic eddy field. The consistency found between statistics derived from icebergs, drifters, and current meters confirms this (Garrett et al., 1985).

1.6 POSITION PREDICTION

In the case of iceberg trajectory prediction we are concerned with position rather than velocity predictions. In considering two-dimensional motion we may want to include the effect of inertial oscillations which affect each component of autocorrelation as well as the cross-correlation. Finally we must consider the effect of observational errors in past iceberg positions used in the prediction scheme.

Garrett et al. (1985) showed that integrating the velocity prediction equations in time results in an equivalent optimum prediction for position, so that for the exponential autocorrelation discussed earlier we obtain $\alpha = \frac{1}{\gamma}(1-e^{-\gamma t})$. Two dimensionality is easily considered by denoting position and velocity vectors as complex quantities $z = x + iy$ and $w = u + iv$. The autocorrelation function is then replaced by $Q(\tau) = R(\tau) + iC(\tau)$ where $C(\tau)$ is the cross-component correlation and we have assumed isotropy ($R_u(\tau) = R_v(\tau)$); isotropy is a reasonable first assumption for the Labrador Shelf, and, as will be seen, is better for the Grand Banks. Inertial oscillations superimposed on an exponential correlation function are allowed by modelling $Q(\tau)$ as:

$$Q(\tau) = A_1 e^{-\gamma_1 \tau} + A_2 e^{-(\gamma_2 + if)\tau} \quad (1.6)$$

where γ_1^{-1} is the exponential decorrelation time,

γ_2^{-1} is the decorrelation time for inertial oscillations, and

f is the Coriolis frequency.

The coefficients A_1 and A_2 denote the relative proportion of velocity variance attributable to the exponential and oscillatory components of the correlation function. For a noiseless data set $A_1 + A_2$ equals 1 because by definition R(0) is equal to 1.

In the case of trajectories observed with some degree of error, two successive velocity estimates, w(t) and w(t+τ), are always likely to be different even as τ tends to zero. This difference results in an observed autocorrelation function R_o which tends to a value less than unity for $\tau \rightarrow 0$. For iceberg trajectory prediction where past velocity estimates are based on successive position observations at constant time intervals, Δt, the observed autocorrelation function is given by:

$$Q_o(\Delta t) = A\,R(\Delta t) - \frac{1}{2}(1 - A), \text{ for the first lag, and}$$

$$Q_o(n\Delta t) = A\,Q(n\Delta t), \text{ for subsequent lags, } n>1. \quad (1.7)$$

This formulation can be readily generalized to the case of non constant lags. In equation 1.6 we therefore have, for large lags and noisy data, $A_1 + A_2 = A$.

The coefficient A is given by:

$$A = \frac{\overline{u^2}}{\overline{u^2} + \overline{\epsilon^2_u}} = \frac{\overline{v^2}}{\overline{v^2} + \overline{\epsilon^2_v}} \qquad (1.8)$$

and denotes the proportion of observed variance, $\overline{u^2} + \overline{\epsilon^2_u}$, not attributable to observational noise, $\overline{\epsilon^2_u}$.

Another effect of observational noise is that substituting $A\ e^{-\gamma\tau}$ for $R(\tau)$ in equation 1.4 we find that the coefficients α'_i for i>1 are no longer zero. This result means that, even for an exponential autocorrelation function, a multi-term predictor can be obtained. Clearly the inclusion of several previous observations will reduce the effect of error, however, Garrett et al. (1985) have shown that, for practical purposes, on the Labrador Shelf, the increased accuracy from using many past iceberg observations is marginal, and a one-term predictor is adequate. More attention will be given to observational noise specifically in the context of the Grand Banks, in Section 3.2 and Appendices B and C.

CHAPTER 2 DATA ANALYSIS FOR THE GRAND BANKS

Several parameters in the general model just described are of a site-specific nature. This chapter describes the data analysis undertaken for the Grand Banks to estimate the statistical parameters required by the model.

2.1 DATA REVIEW

Existing reports and data analyses were first examined for a preliminary assessment of drift statistics on the Grand Banks. This review yielded mostly a qualitative description, and a further analysis of 388 iceberg tracks and several current meter records was undertaken better to define drift characteristics on the Grand Banks.

Petrie and Isenor (1984) and Isenor and Petrie (1985) have analysed means and variances of 39 satellite drifters reporting positions, over all seasons, from the Grand Banks and Labrador Current. Birch (1985) has analysed five, satellite-tracked drifters deployed in the Hibernia region in November 1984. Some of their results, pertinent to the region of interest, are shown in Table 2.1. The error associated with each drifter satellite fix is in the order of 4 km, so that the typical sampling rate of 8 hours implies a velocity error of 0.10 m/s in the daily averages they have used to compute drift statistics. In addition to positional and sampling error, the statistics are also contaminated by the effects of wind, inertial waves, and tides. These errors, combined with the relatively few buoy days in each 1^ox 1^o square, imply that the estimated statistics are qualitative at best.

TABLE 2.1

Summary of velocities from Petrie and Isenor (1984) and Birch (1985)

Source description	$\overline{u}$ m/s	$\overline{v}$ m/s	$(\overline{u'^2})^{.5}$ m/s	$(\overline{v'^2})^{.5}$ m/s	Number of buoy days
Petrie and Isenor: Drifter data over all seasons, grouped into $1^o x 1^o$ cells, centred on:					
$46^o30'N$ $48^o30'W$	-0.22	0.30	0.27	0.36	3
$46^o30'N$ $49^o30'W$	-0.08	-0.03	0.26	0.07	3
$47^o30'N$ $47^o30'W$	0.09	-0.13	0.18	0.22	83
$47^o30'N$ $48^o30'W$	0.11	-0.05	0.16	0.11	45
Birch: Drifters released near Hibernia on the same day, grouped for the Grand Banks and the Labrador Current.					
Grand Banks					
drifter # 5419	0.02	-0.07	0.12	0.13	
drifter # 5420	0.03	-0.08	0.14	0.16	
drifter # 5421	0.04	-0.05	0.12	0.13	
drifter # 5422	0.05	-0.06	0.11	0.13	
drifter # 5428	0.05	-0.04	0.12	0.12	
				total days ~ 25	
Labrador Current					
drifter # 5419	-0.04	-0.45	0.16	0.22	
drifter # 5420	-0.07	-0.48	0.12	0.22	
drifter # 5421	-0.15	-0.42	0.20	0.27	
drifter # 5422	-0.15	-0.42	0.15	0.26	
drifter # 5428	-0.07	0.33	0.13	0.23	
				total days ~ 8	

In their reports they show a strong mean flow parallel to the 200-m isobath. On the Grand Banks the mean speed is typically 0.05 m/s, whereas in the Labrador Current the mean speed ranges from 0.25 to 0.40 m/s. On the Grand Banks the standard deviation in drift velocity is in the order of 0.13 m/s, whereas in the Labrador Current it lies between 0.16 m/s and 0.24 m/s.

Several current meter records are also available for the top 30 m of the water column near Hibernia ($48^{o}50$'N, $46^{o}45$'W). Using the data collected from two, vector-averaging current meters at 20 m depth, Petrie[1] obtained a mean current of 0.017 m/s with total standard deviation of 0.13 m/s, which confirms the drifter results. The low Reynolds stresses, (momentum transfer arising from the cross-correlation of velocity components in a shear flow), at Hibernia imply small local shear. We may therefore expect that the shear resulting from the Labrador Current is limited to a narrow band near the edge of the shelf and and has little effect at Hibernia.

Isenor and Petrie (1985) and Birch (1985) present estimates of the Lagrangian velocity autocorrelation. Those presented by Birch (1985) for the Grand Banks indicate the presence of wave motions of period 4 days. The autocorrelations presented by Isenor and Petrie (1985) are representative of the Grand Banks over April-November 1981, and show oscillations with time

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[1] B. Petrie, Bedford Institute of Oceanography, personal communication, 1985.

scales near 5 days. The overall decorrelation time scales vary between 20 and 40 hours. Large sampling errors in both data sets are evidenced by the convergence of the autocorrelation function to a value below unity at small time lags.

2.2 RAW ICEBERG VELOCITY STATISTICS

Iceberg trajectory and surface wind data for 1984 and 1985 were acquired from the east coast operators. The trajectory data were obtained by visual and radar fixes from vessels, oil rigs, and aircraft; most observations were from the region (46 - 48^{o}N, 47 - 49^{o}W) generally to the east of the Hibernia site.

In estimating velocities we have excluded the trajectories of icebergs that are either grounded (immobile for several subsequent fixes), or under tow, or where successive positional fixes are more than 6 hours apart. Histograms of the velocity estimates were then obtained for the Labrador Current and Hibernia regions and outliers (velocity estimates beyond 4 times the standard deviation, or about 2 knots) were removed. As shown in Table 2.2, this process led to a reduction (by about 80%) in the number of usable tracks and velocity estimates. Figure 2.1 shows the trajectories retained for 1984 and 1985. The spatial distribution of the data differs for the two years. The wind data used were the 6-hourly gridded surface (~10 m) analysis products used operationally on the Grand Banks for 1984 and 1985.

TABLE 2.2

Summary of iceberg velocity estimates

Observations	Raw velocity estimates	Usable velocity estimates
1984 observations from		
vessels	2620	803
rigs	1607	332
aircraft	1247	158
others	0	0
total	5474	1293
1985 observations from		
vessels	2254	430
rigs	748	114
aircraft	589	7
others	748	14
total	4339	563

Only icebergs with two or more position fixes are used to obtain raw velocity estimates. From these all estimates for bergs which were towed, grounded, or based on fixes more than 6 hours apart were eliminated. Outliers are further removed from the remaining population to obtain our final selection of usable iceberg tracks and velocity estimates.

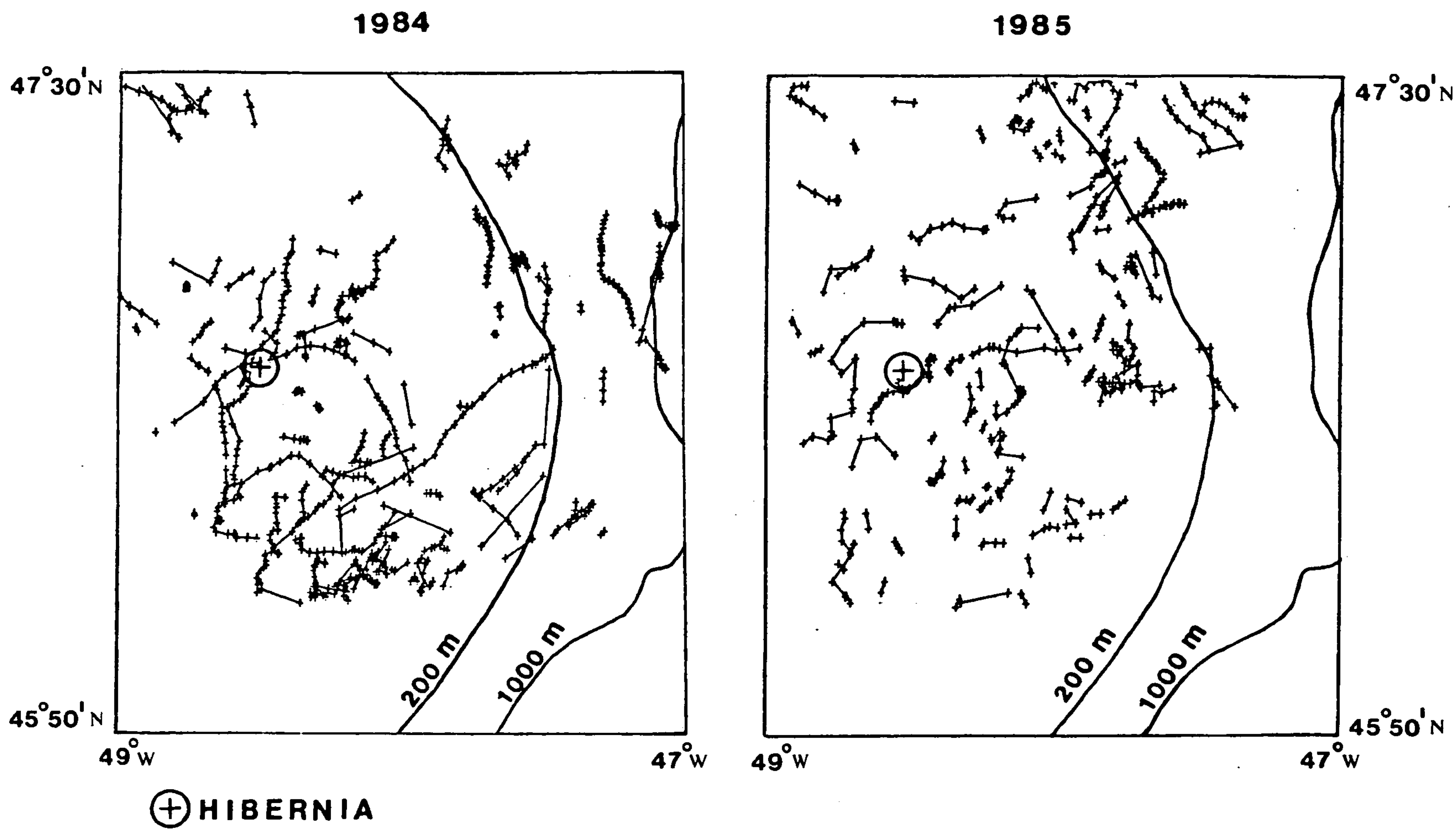


Figure 2.1. Observed iceberg trajectories used for analysis.

After editing, the global mean and standard deviation of iceberg velocities were computed and are shown in Table 2.3. Iceberg velocity estimates were also grouped into subquadrants about the Hibernia site at radial intervals of 50 km. Component standard deviations (northward and eastward) and means of velocity were then determined for each area. To increase reliability, an average of the 1984 and 1985 results, weighed by the number of observations in each year, was made to obtain the estimates presented in Figure 2.2.

The mean velocities suggest a clockwise flow about the Hibernia site with some exceptions (flow for the southeast (50, 100) km subquadrant opposes the expected southward flow of the Labrador Current). The component velocity variance is apparently larger (0.20 to 0.30 m/s) than that reported by Isenor and Petrie (1985); however, one must note that we have not yet allowed for tides, inertial oscillations, and the high noise level in our data, and Isenor and Petrie (1985) have effectively filtered all high-frequency motion through daily averaging.

2.3 TIDES

Garrett et al. (1985) found very good correspondence between the tidal motion of icebergs and current meter observations. For the Grand Banks, tidal velocities are known to depend significantly on local water depth; however, the small number of available iceberg velocity observations only permits a global estimate of tidal constants. These estimates agree qualitatively with

TABLE 2.3

Mean and standard deviation of raw and residual iceberg drift (m/s)

Source	$\overline{u}$	$\overline{v}$	$(\overline{u'^2})^{0.5}$	$(\overline{v'^2})^{0.5}$
1984 raw	0.00	-0.03	0.27	0.32
1985 raw	0.08	-0.01	0.27	0.28
1984 residual	0.00	-0.11	0.24	0.28
1985 residual	0.01	-0.05	0.25	0.26

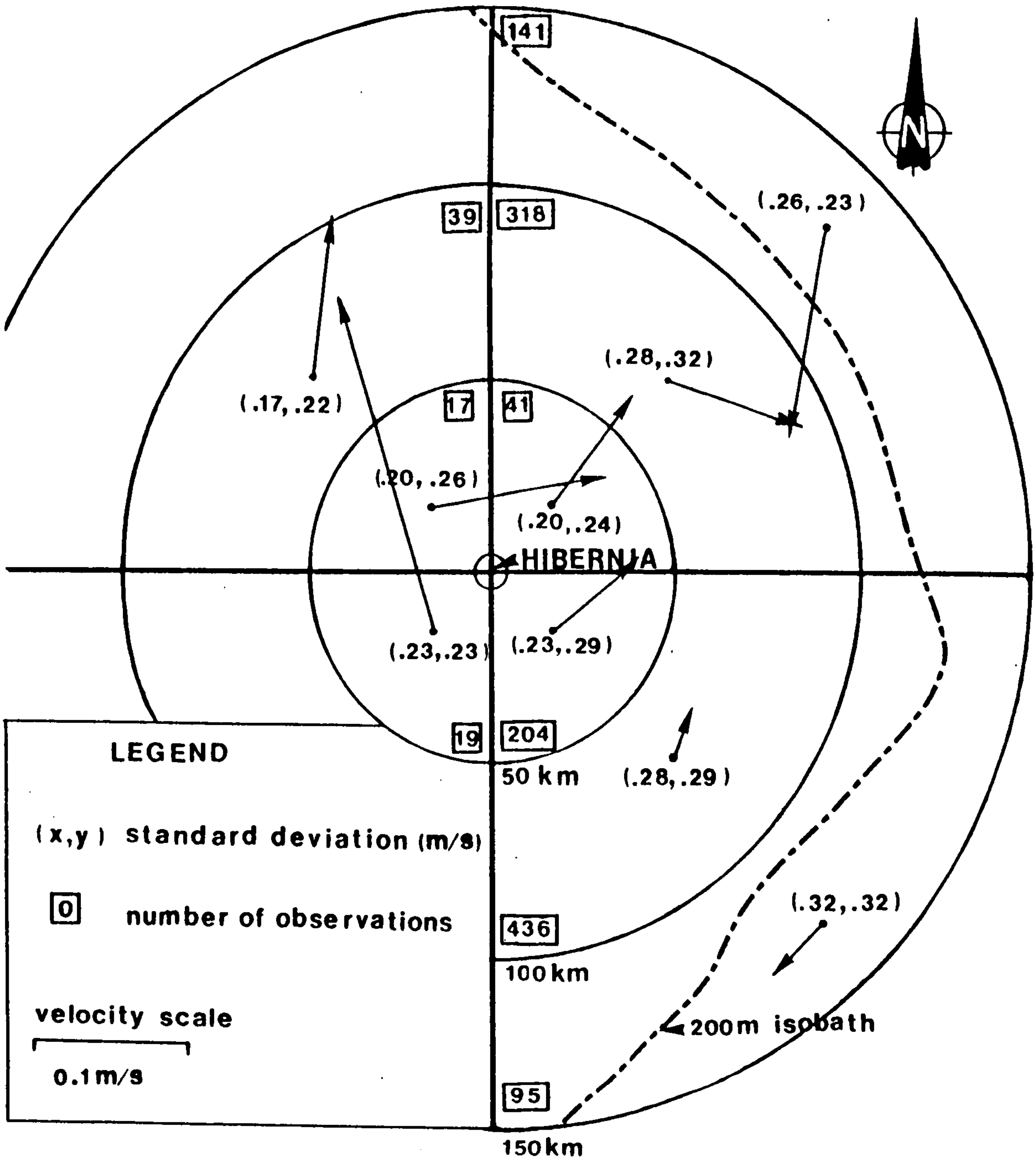


Figure 2.2. Spatial distribution of the mean and variance of raw iceberg drift.

current meter observations but are plagued by the effects of observational noise and spatial variability. It was therefore decided to use tidal constants derived from numerical experiments (de Margerie and Lank, 1986) in subsequent analysis. These numerical experiments show good agreement with observations and describe tidal flow with a spatial resolution in the order of 7 km. For the present purpose this information was reduced to a 20' by 20' grid covering the eastern Grand Banks. As an example the constants for the M2 constituent are given in Table 2.4. The M2, N2, S2, O1, and K1 constituents account for most of the tidal variance and are included in our analysis. Tides have been found to account for up to 50% of the iceberg velocity variance on the shallowest parts of the Grand Banks (Petrie, 1982). Near the edges of the banks, the decrease in tidal signal with increasing water depth, combined with the higher overall variance, reduces the tidal effects to an insignificant level.

2.4 ICEBERG/WIND VELOCITY CORRELATIONS

An important part of the iceberg motion can be attributed to wind, either directly through drag above the water line, or indirectly by the currents that the wind generates. To determine the influence of wind we will analyse the iceberg motion with the removal of the tidal signal. Later the wind signal itself will be removed to examine the statistics of the residual drift.

TABLE 2.4

Tidal constants for the M2 constituent

U component amplitude (phase) in m/s (degrees GMT)

	$49^{o}00'$–$48^{o}40'$	$48^{o}40'$–$48^{o}20'$	$48^{o}20'$–$48^{o}00'$	$48^{o}00'$–$47^{o}40'$	$47^{o}40'$–$47^{o}20'$	$47^{o}20'$–$47^{o}00'$
$47^{o}30'$–$47^{o}10'$	0.059 (59)	0.052 (56)	0.048 (53)	0.043 (51)	0.039 (51)	0.020 (54)
$47^{o}10'$–$46^{o}50'$	0.078 (55)	0.065 (53)	0.058 (51)	0.048 (51)	0.038 (53)	0.014 (59)
$46^{o}50'$–$46^{o}30'$	0.097 (50)	0.078 (48)	0.067 (46)	0.052 (45)	0.034 (46)	0.010 (51)
$46^{o}30'$–$46^{o}10'$	0.115 (45)	0.089 (42)	0.075 (38)	0.057 (33)	0.033 (24)	0.011 (4)
$46^{o}10'$–$45^{o}50'$	0.127 (40)	0.098 (37)	0.080 (34)	0.049 (30)	0.020 (25)	0.017 (18)

V component amplitude (phase) in m/s (degrees GMT)

	$49^{o}00'$–$48^{o}40'$	$48^{o}40'$–$48^{o}20'$	$48^{o}20'$–$48^{o}00'$	$48^{o}00'$–$47^{o}40'$	$47^{o}40'$–$47^{o}20'$	$47^{o}20'$–$47^{o}00'$
$47^{o}30'$–$47^{o}10'$	0.029 (341)	0.022 (327)	0.020 (314)	0.018 (304)	0.015 (298)	0.004 (260)
$47^{o}10'$–$46^{o}50'$	0.044 (327)	0.035 (319)	0.030 (311)	0.023 (305)	0.015 (302)	0.003 (181)
$46^{o}50'$–$46^{o}30'$	0.061 (314)	0.050 (307)	0.042 (301)	0.032 (295)	0.017 (284)	0.007 (179)
$46^{o}30'$–$46^{o}10'$	0.082 (303)	0.066 (297)	0.057 (292)	0.044 (284)	0.026 (269)	0.011 (213)
$46^{o}10'$–$45^{o}50'$	0.100 (297)	0.082 (291)	0.068 (288)	0.041 (285)	0.014 (285)	0.010 (287)

As noted in Chapter 1, the iceberg velocity (after removal of tides), U_b, is assumed to be linearly related to winds by:

$$U_b = U_r + \alpha_w U_w \qquad (2.1)$$

where U_b, U_r, and U_w are the iceberg velocity, the residual velocity, and the wind velocity vectors, respectively, while α_w is a tensor assumed to be constant. The usefulness of the linear response model (equation 2.1) may be examined by considering the cross-correlation of iceberg velocities and winds, defined as:

$$R_{w_i b_j}(\tau) = \overline{U'_{w_i}(t)\, U'_{b_j}(t+\tau)} \;/\; \left(\overline{U'^2_{w_i}}\;\; \overline{U'^2_{b_j}} \right) \qquad (2.2)$$

where primes denote removal of global means (overall average with no regards to spatial variability),

overbars denote mean values,

i and j subscripts range from 1 to 2 to denote x and y directions, and

τ is a time lag.

Substituting equation 2.1 into 2.2 we obtain:

$$R_{w_1 b_1}(\tau) = \left[\alpha_{w_{11}} R_{w_1 w_1}(\tau) + \alpha_{w_{12}} R_{w_1 w_2}(\tau)\right] \beta$$

$$R_{w_2 b_1}(\tau) = \left[\alpha_{w_{11}} R_{w_2 w_1}(\tau) + \alpha_{w_{12}} R_{w_2 w_2}(\tau)\right] \beta \qquad (2.3)$$

where $\beta = \overline{U'^2_{w_i}} \;/\; \overline{U'^2_{b_j}}$, and similar expressions can be derived for $R_{w_2 b_2}$ and $R_{w_1 b_2}$. For an isotropic flow we can show that:

$$R_{w_2 b_2} = R_{w_1 b_1} \text{ and,}$$

$$R_{w_1 b_2} = -R_{w_2 b_1}.$$

Results for the 1984 data are shown in Figure 2.3. The reasonably symmetric peak in $R_{w_1b_1}$ (and $R_{w_2b_2}$) implies that $\alpha_{w_{12}} R_{w_1b_2}$ (and $\alpha_{w_{21}} R_{w_2b_1}$) are relatively small because $R_{w_1b_2}$ must be an odd function of τ, assuming an isotropic wind field. Furthermore, the fact that $R_{w_2b_1}$ (and $R_{w_1b_2}$) are near zero indicates that $\alpha_{w_{11}} R_{w_2b_1}$, $\alpha_{w_{12}} R_{w_2b_1}$, etc. must be near zero, so that $\alpha_{w_{11}} \simeq \alpha_{w_{22}}$, $\alpha_{w_{12}} \simeq \alpha_{w_{21}} \simeq 0$ and $R_{w_1b_2} \simeq R_{w_2b_1} \simeq 0$. The results reinforce the model assumption of isotropic response and agree with the simple downwind drift found by Garrett et al. (1985) on the Labrador Shelf.

Results for the 1985 data are presented in Figure 2.4. The $R_{w_1b_1}$ and $R_{w_2b_2}$ are consistent with the 1984 data; however, the cross-component correlations show a consistent non-zero signal which mirrors the co-component relationship. This result implies again that $R_{w_{12}} \simeq R_{w_{21}} \simeq 0$ but $\alpha_{w_{12}}$ and $\alpha_{w_{21}}$, although smaller than $\alpha_{w_{11}}$ and $\alpha_{w_{22}}$, are non zero. It should be noted that the response is non-isotropic and inconsistent with Coriolis rotation, because north winds cause a component of drift to the left and east winds cause a component of drift to the right.

The 1985 response is difficult to explain without further study, but may be associated with the different location of most data in that year (more to the north east). In the absence of a better understanding for this phenonema, the 1985 data will be treated as anomalous, and for the sake of simplicity we will set $\alpha_{w_{21}} = \alpha_{w_{12}} = 0$ and $\alpha_{w_{11}} = \alpha_{w_{22}}$, as indicated by the 1984 data. The tensor α_w is then equivalent to a simple scalar coefficient. For 1984 and 1985, α_w is estimated at 0.016 and 0.020, respectively. The average value of

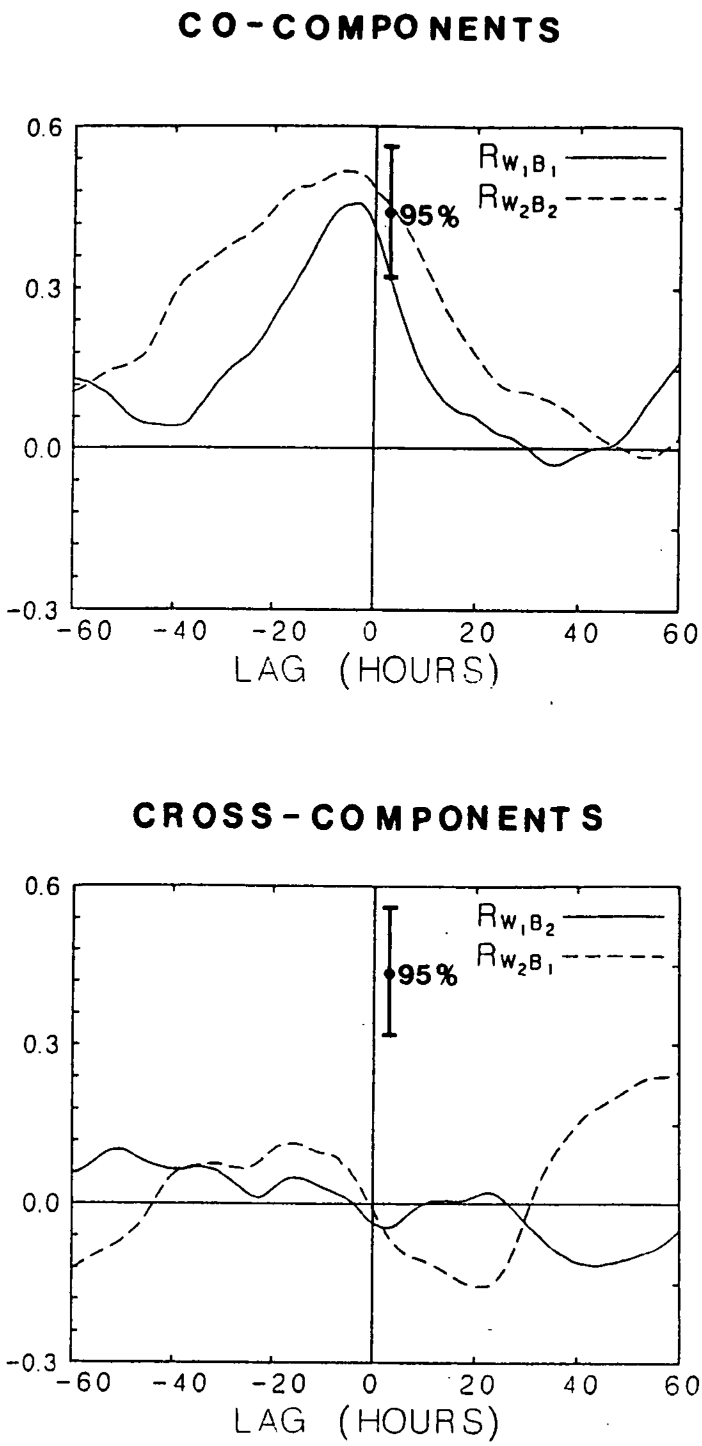


Figure 2.3. Wind/iceberg velocity cross-correlation function for 1984.

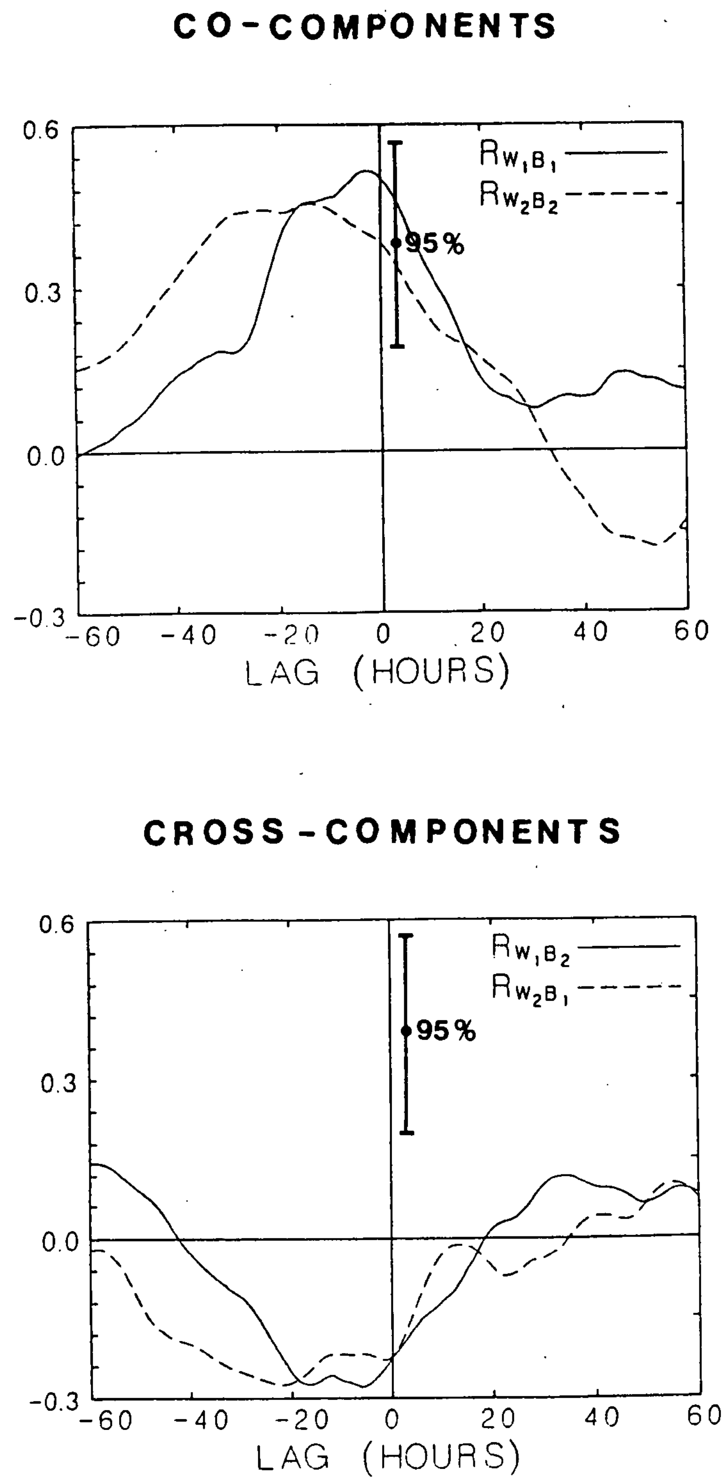


Figure 2.4. Wind/iceberg velocity cross-correlation function for 1985.

0.018 is the same as that found by Garrett et al. (1985) from an extensive analysis of iceberg observations on the Labrador Shelf, and will therefore be used for our Grand Banks iceberg prediction model as well as in removing the wind-driven component of motion in the analysis to follow. Removal of the tidal and wind-driven component reduces the global standard deviation for each velocity component from 0.30 m/s to 0.26 m/s.

2.5 ICEBERG VELOCITY AUTOCORRELATIONS

Denoting u and v to be the two components of residual iceberg velocity U_r (tide, wind, and mean flow removed), we define:

$$R_u(\tau) = \overline{u(t)\ u(t+\tau)} \ / \ \overline{u^2} \tag{2.4}$$

$$R_v(\tau) = \overline{v(t)\ v(t+\tau)} \ / \ \overline{v^2} \tag{2.5}$$

to be the Lagrangian component velocity correlations. Estimates R_u and R_v were obtained by forming the product of all lagged pairs of residual velocity for each iceberg followed by an average over all pairs. The products were then averaged and normalized for each lag. In performing the averages we have, because of the lack of data, ignored the spatial variability of statistics so that the overbars and primes in equations 2.4 and 2.5 denote computation or removal of global means. Because of the spatial variability of the statistics the correlations presented may be somewhat contaminated.

As discussed in Appendix B, positional error results in a scaling of the autocorrelation function, which for small lags tends to A R(0) - 0.5(1 - A) where

$$A = \overline{u_b^2} \ / \ (\overline{u_b^2} + 2\frac{\overline{\epsilon^2}}{\Delta t^2}) \tag{2.6}$$

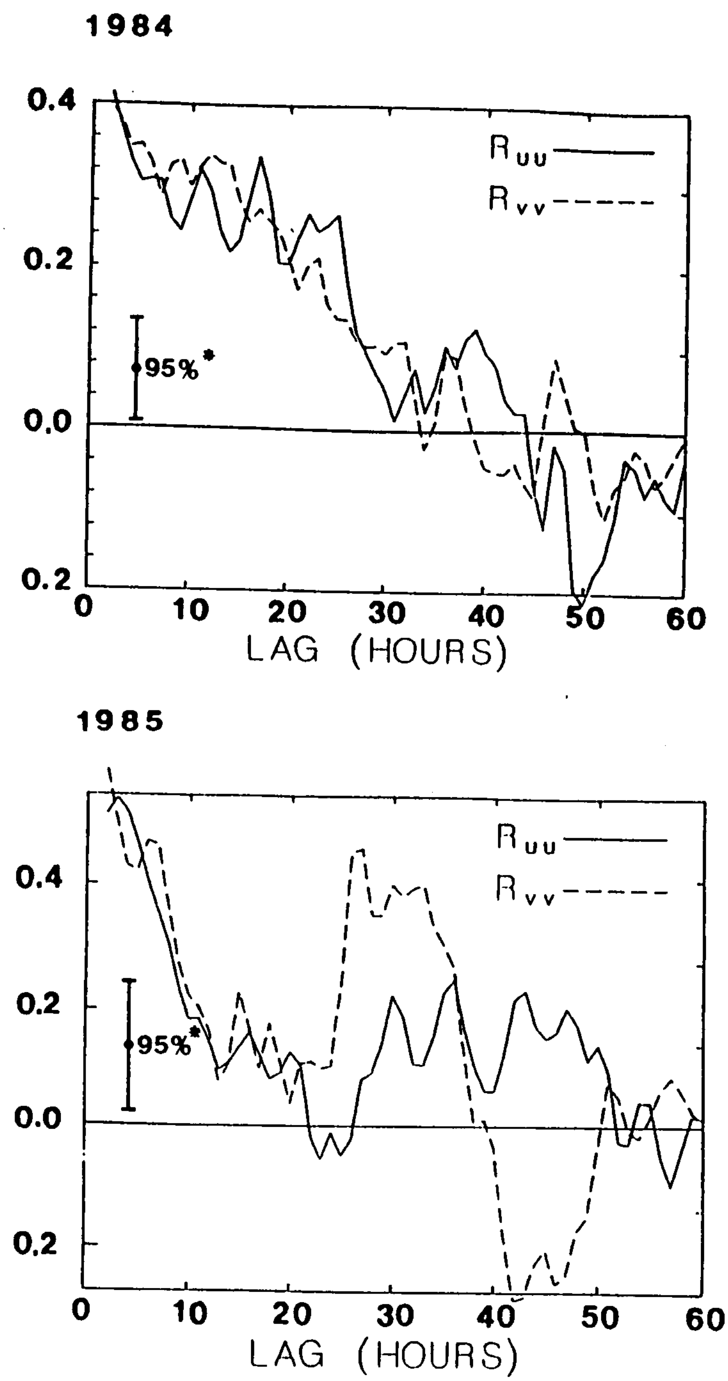
where $\overline{u_b^2}$ is the true iceberg component velocity variance,

$\overline{\epsilon^2}$ is the variance in position error, and

Δt is the observational time interval,

with a similar expression for the v component. At lags greater than one the autocorrelation is simply scaled by A. At small lags the true autocorrelation function should tend to unity, however, the 1984 and 1985 data show values of 0.40 and 0.55, respectively. Corresponding A's are 0.6 and 0.7, with a weighted average of 0.65. With the residual variance of Section 2.4 this implies a true standard deviation for each velocity component of 0.20 m/s and using a typical observational interval of 2 hours the positional error is in the order of 750 m. Compared to the standard deviations for the raw data, the values with tides, winds, and noise removed are about 50% lower.

With the effect of noise so determined, the estimates of correlation were rescaled by A and then smoothed with a running mean with weights of 0.25, 0.50, and 0.25. The results for 1984 and 1985 are shown in Figure 2.5. All autocorrelation functions exhibit low-frequency oscillations. The 1984 data shows wavelike motion at a period of about 180 hours (estimated as four times the first zero crossing period) in both velocity components. This result is very similar to those of Birch (1985), even though his drifters show oscillations at about 160 hours. The 1985 data are considerably more complex and difficult to interpret. The difference in correlation for the two velocity components indicates considerable anisotropy, with the v (north-south) component showing a strong diurnal signal. The anomaly of the



* Confidence intervals are for small lags; due to the paucity of long tracks, confidence intervals grow rapidly with lag.

Figure 2.5. Iceberg velocity autocorrelation for 1984 and 1985.

1985 data may result from the different location of most tracks which are nearer to the shelf edge. The strong diurnal signal in the v component corresponds to motion along the isobaths and could be the signature of diurnal shelf trapped waves[1].

The autocorrelation functions were plotted on a logarithmic scale and were fitted visually by straight lines, the slopes of which gave a representative decorrelation time. All data fit reasonably with a $\gamma^{-1} = 25$ hours for the Lagrangian time scales, although the uncertainty for the 1985 data is considerable. The corresponding model autocorrelation function is given by:

$$R(\tau) = e^{-\gamma \tau}, \text{ with } \gamma^{-1} = 25 \text{ hours}$$

fitting a model which includes the wavelike motion exhibited in the 1984 data yields

$$R(\tau) = e^{-\gamma \tau} \cos(2\pi \omega \tau), \text{ with } \gamma^{-1} = 45 \text{ hours and } \omega^{-1} = 180 \text{ hours.}$$

In view of the long time scale of the oscillatory motion compared to the time of concern in iceberg forecasting, it can be concluded that a simple exponential will adequately parameterize the autocorrelation function.

2.6 INERTIAL WAVES

Both current meter and iceberg estimates of variance include a contribution from inertial waves. At the latitude of Hibernia the period of

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[1] B. Petrie, Bedford Institute of Oceanography, personal communication, 1985.

inertial motion is about 17 hours. The signature of inertial waves in the autocorrelation function can be seen as an oscillation at the inertial period especially in the cross-correlations of the u and v components. Current meter data for the Grand Banks clearly exhibit oscillation in the inertial band. An analysis of current data was undertaken for an instrument moored 20 m deep using a wave-insensitive instrument near Hibernia ($46^{o}51'25''N$, $48^{o}45'36''W$), in spring of 1982. The analysis consisted of evaluating variances and autocorrelation functions (Figure 2.6). Assuming that inertial oscillations are independent of other motion, the observed autocorrelations can be modelled by:

$$R(\tau) = A_1 \cos(f\ \tau)\ e^{-\gamma_2 \tau} + A_2\ R'(\tau) \qquad (2.7)$$

where $R'(\tau)$ is the part of the signal not resulting from inertial waves,

A_1 and A_2 are the proportions of the total variance attributable to inertial and other motions respectively,

γ_2^{-1} is the inertial decorrelation time, and

f is the Coriolis frequency.

The parameter γ_2^{-1} and the proportion of variance attributable to inertial waves were evaluated as 60 hours and 0.2, corresponding to an rms component velocity of 0.07 m/s.

As seen previously in Figure 2.5 the iceberg autocorrelations do not exhibit inertial oscillations, confirming that the contribution of inertial waves to the iceberg motion is small and is probably obliterated by the noise level in the data.

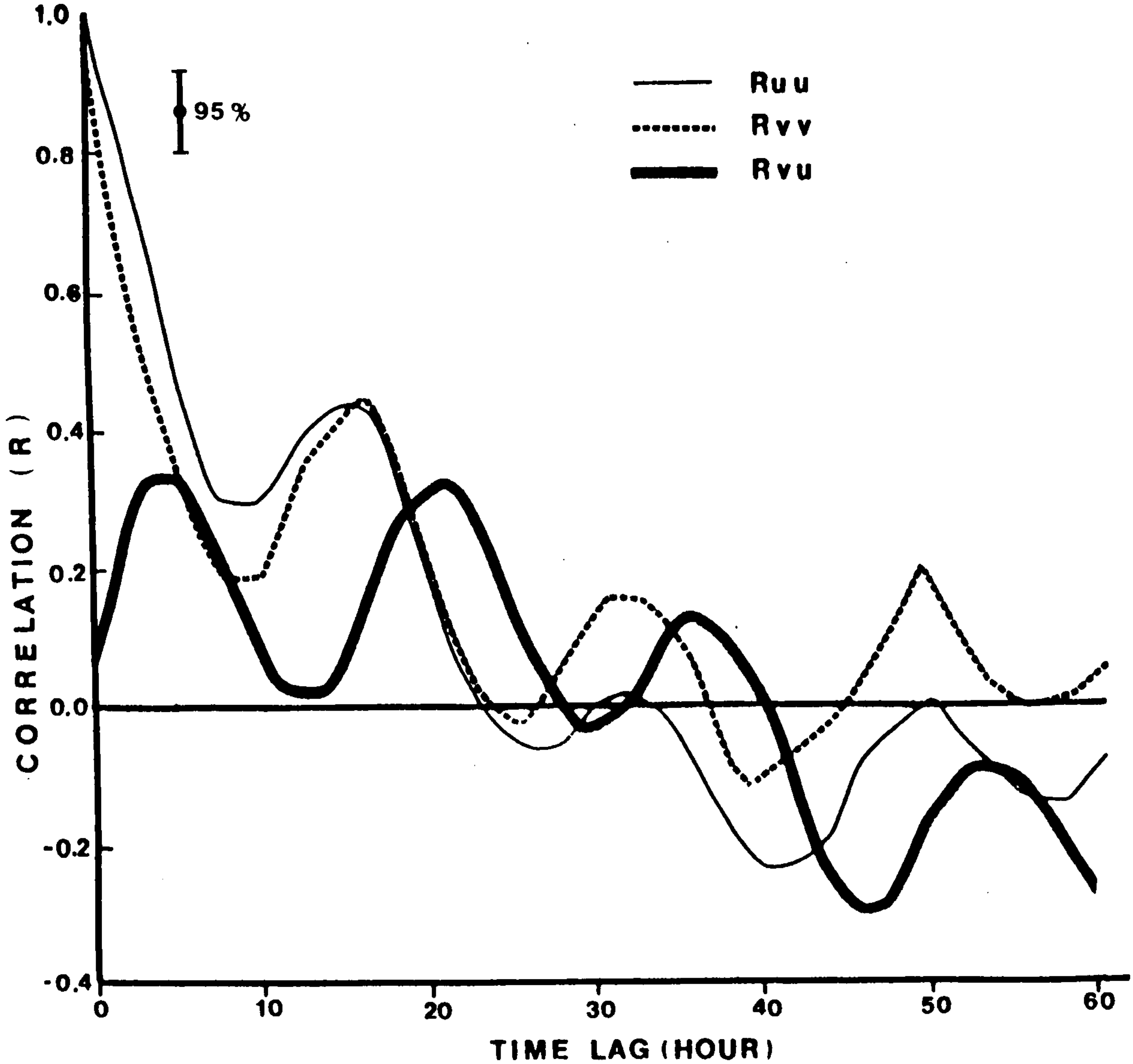


Figure 2.6. Eulerian velocity auto- and cross-correlation functions, obtained from a current meter at 46°51'25"N and 48°45'36"W in April and May 1982.

2.7 SPATIAL DISTRIBUTION OF RESIDUAL VELOCITIES

To determine the spatial variability of the mean and standard deviation of iceberg velocities, observed velocity estimates were grouped into cells of a 20' by 20' grid within the study area. The effect of tides, winds, and noise level discussed earlier were removed and the estimates of Figure 2.7 were obtained. Standard deviations showed no systematic differences in the x and y components and the radius of circles shown in the figure correspond to the mean of the x and the y variances (the parameter c in Chapter 1 is equal to 1.42 times this value). It should be pointed out that the number of observations indicated in each cell does not represent the available degrees of freedom, because consecutive velocity estimates are not independent. Based on the mean observational interval, the Lagrangian decorrelation time scale and the typical number of observations per track, we estimate that the number of degrees of freedom is about one-fifth the number of observations. Using this figure we find that most of the mean velocity estimates previously presented in Figure 2.6 are only marginally significant at the 63% level.

We can nevertheless infer a mean flow of 0.20 to 0.35 m/s along the 200-m isobath. A maximum in variances occurs near the edge of the bank at $46^{o}30'$N with standard deviations of over 0.30 m/s. The standard deviation in the Hibernia region varies between 0.10 m/s and 0.17 m/s and agrees with estimates from current meter data presented earlier.

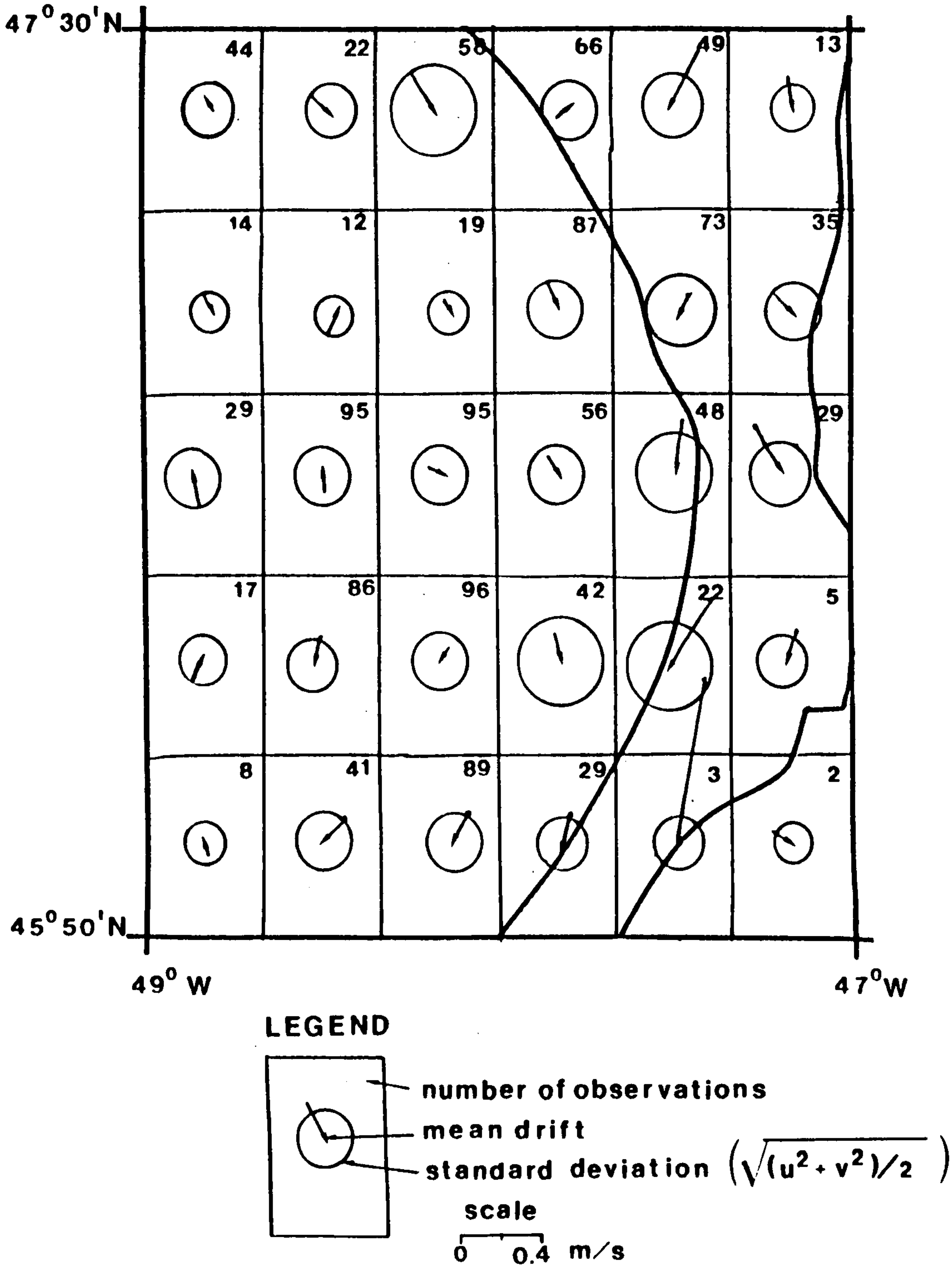


Figure 2.7. Spatial distribution of mean and standard deviation for residual iceberg velocities.

CHAPTER 3 MODEL CONSIDERATIONS FOR THE GRAND BANKS

Several enhancements of the original model (Garrett et al., 1985) were necessary for the Grand Banks implementation, because of the important spatial variability of iceberg statistics, the accuracy of observations, and the mode of operation of oil rigs on the Grand Banks. These special considerations, together with model verification, are discussed below.

The properties of observed iceberg tracks presented for the Grand Banks in Chapter 2 differ in several respects from the Labrador Shelf data used in Garrett et al. (1985). As a result a few modelling procedures have been revised.

A main feature of iceberg drift on the Grand Banks is spatial variability in both variance and mean flow. To accommodate this, the operational model uses tidal currents, mean flows, and variances, defined on a grid. Iceberg trajectories and confidence intervals are then predicted using values corresponding to the location of the iceberg.

The 1984 and 1985 Grand Banks data also showed greater noise contamination than those of Garrett et al., which has implications for the confidence attributable to iceberg prediction, because it increases the uncertainty in the initial iceberg velocity, U(0), needed for the statistical component of the model. To decrease the effect of observational noise on predictions, two possible schemes are considered. The first uses several velocity estimates as predictors, whereas the second uses one velocity estimate based on observations taken a longer time apart. In general both will give similar but small improvements in prediction (about 10% reduction in residual error), because a large part of the uncertainty in iceberg position is attributable to the random nature of iceberg motion itself. In any case the second alternative is computationally simple and is implemented in the model. The model actually allows variable time intervals between position observations, choosing the optimum interval when more than two observations are available. In some cases, this selection can result in a substantial improvement in prediction.

The simple probability calculation proposed by Garrett et al. (1985) to calculate the chance of intersection of the iceberg trajectory with a target of known dimension was enhanced to include the effect of all velocity components (tidal, wind driven, etc.) as the iceberg approaches the target. The model computes the probability at hourly intervals during a prediction, summing them to obtain the total probability of impact.

General considerations indicate that the use of rig-observed currents could be used to improve predictions, especially in view of the large apparent observational error in iceberg sightings. The improvement, however, is likely to be small, but has not been investigated in this study. At present the model cannot accommodate rig-observed currents in the prediction scheme.

Model verification shows good agreement between predicted and observed tracks. Deviations between predicted and observed positions, although large, are within the confidence intervals computed by the model.

3.1 INCLUSION OF SPATIALLY VARYING VELOCITIES

In modelling iceberg drift in our study area, one must acknowledge the spatial variability of means and standard deviations which may cause changes of a factor of 5 and 3 respectively. The direct implication of this variability is obvious if one considers that, near the edge of the Grand Banks, the mean flow can be the principal component of motion and that its neglect would cause major errors in the prediction of iceberg position. The estimate of confidence intervals for predictions is directly proportional to the standard deviation of velocity so that neglecting spatial variability could result in serious misinterpretation of model results. To include these first order effects in the prediction scheme we have assessed the mean and standard deviation of velocities in 20' by 20' cells within the study area (Figure 3.1). These values are based on the results of the previous section, with some modifications. The standard deviations are the same as those observed, except on the shelf in cells where fewer than 20 observations were available. In these cases, new estimates were computed to include the data from adjoining cells. Mean flows on the shelf are as observed, again with the exception noted above. Because the flows on the shelf are relatively small, the uncertainty in these estimates would not significantly affect predictions. Near the shelf edge, on the other hand, the Labrador Current causes an

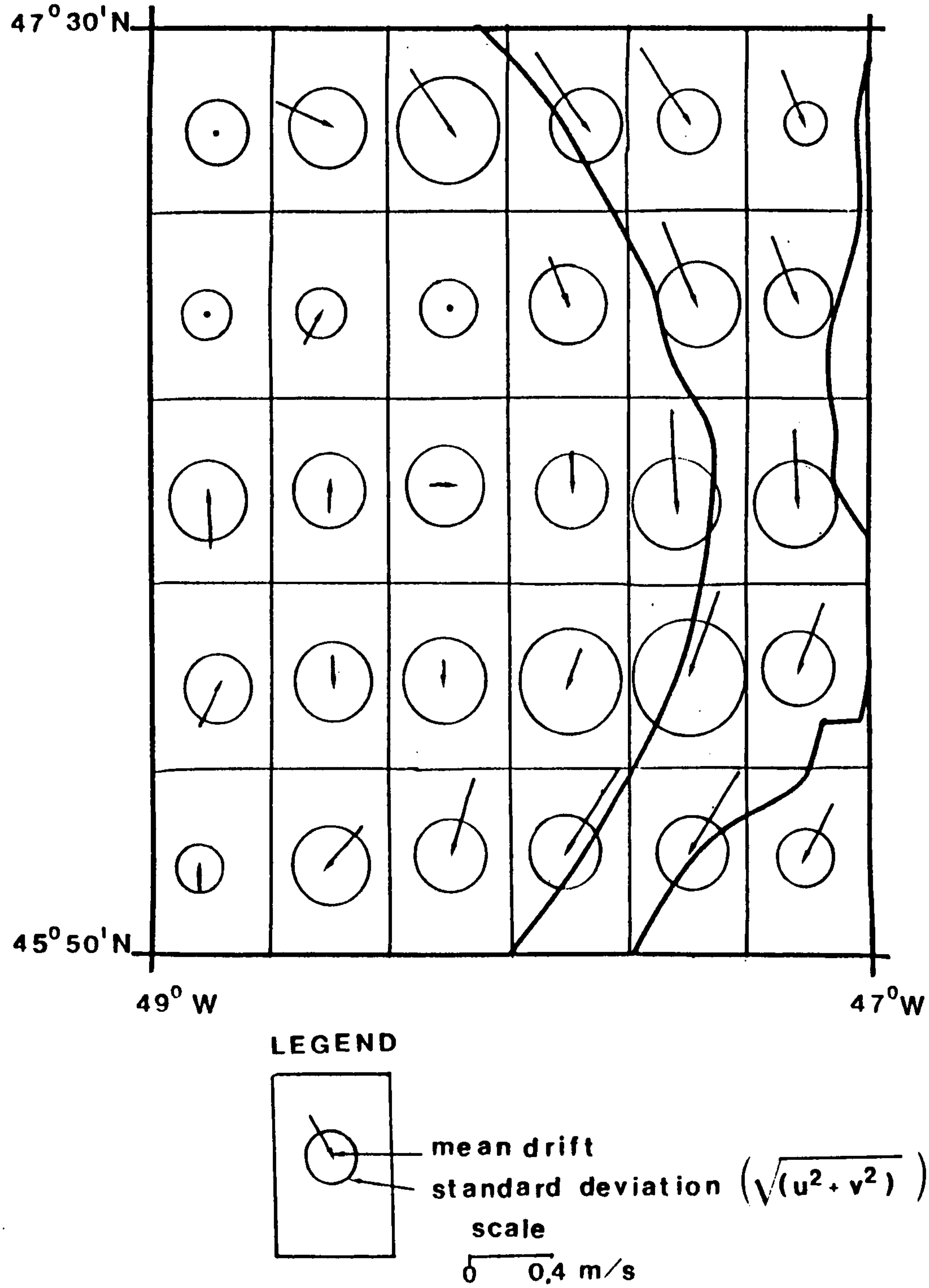


Figure 3.1. Gridded mean and standard deviation of residual iceberg drift, used for predicting iceberg trajectories.

important mean drift component. To improve the statistical reliability of the mean flow estimates in this region, the shelf edge was bounded by circular arcs about Hibernia (see Figure 2.2), and a mean flow of 0.25 m/s along the isobath was computed. This pattern is consistent with other studies (Petrie and Isenor, 1984), and was used to estimate the mean flow near the shelf edge.

For iceberg predictions the model will use values of mean flow $\overline{U}$ and variance c^2 corresponding to the cell where the iceberg is located. It should be noted that spatial gradients in mean flow and variance can lead to higher order effects such as shear dispersion and an added mean transport in a manner analogous to turbulent entrainment. These are discussed in more detail in Appendix A and are shown to have negligible effect.

3.2 OPTIMUM CHOICE OF PREDICTORS

As discussed in Section 1.5, in the presence of observational noise in past iceberg observations there may be an advantage in using several velocity estimates in the statistical prediction equation. Alternatively one may consider combining several past velocity estimates into a mean value to increase the confidence of a single predictor. Although the second alternative is computationally more attractive, the former may actually provide better accuracy. Both possibilities are considered in Appendices B and C respectively. The conclusions depend essentially on the observational time interval Δt, because, as indicated in the following, the noise level in velocity estimates used as predictors depends on this interval.

In the case where we have a set of velocity estimates obtained from successive position observations, taken Δt apart, and each with an uncorrelated error, ϵ, we have in one dimension, and with mean flow removed:

$$u_{o\,n} = (x_{n+1} - x_n + \epsilon_{n+1} + \epsilon_n) / \Delta t \qquad (3.1)$$

The observed velocity variance is then:

$$\overline{u_o^2} = \overline{u^2} + 2\,\overline{\epsilon^2} / \Delta t^2$$

where $\overline{u_o^2}$ is the observed variance,

$\overline{u^2}$ is the true variance, and

$\overline{\epsilon^2}$ is the variance of positional errors.

Substituting $2\,\overline{\epsilon^2} / \Delta t^2$ for $\overline{\epsilon_u^2}$ in equation 1.8 gives $A = \left[1 + \dfrac{2\overline{\epsilon^2}}{\overline{u^2}\,\Delta t^2} \right]^{-1}$.

Therefore the factor A, representing the relative importance of observational noise in the prediction, depends on Δt. For the 1984 and 1985 data sets, A equals 0.65 with a representative Δt of 2 hours. If the observational interval would have been half or double this value (1 or 4 hours), A would be equal to 0.34 or 0.88, with drastic implications on the confidence of predictions.

In Appendix B it is shown that for observational errors and intervals similar to those of the 1984 and 1985 data, the advantage from using several predictors is marginal, typically reducing the prediction uncertainty by 5% or 10%. For the case of one hour intervals the advantage of using several predictors is much greater. Appendix C considers the possibility of simply increasing the observational interval to increase A, and therefore increase the confidence in a single term prediction for times which are small when compared to the Lagrangian time scale. Doubling the observational interval is equivalent to using the average velocity over the first two intervals and equation 3.1 becomes:

$$u_1 = (x_3 - x_1 + \epsilon_3 + \epsilon_1) / 2\Delta t \qquad (3.2)$$

The increase in accuracy using this method is almost the same as using several predictors; however, computations are much simpler. Obviously there must be a limit to which we can increase the observational interval, since if it is very large the resulting velocity estimate will bear little relevance to its present or future velocity. This limit depends on the decorrelation time scale, γ, the residual velocity variance, c^2, and the positional error, ϵ. From the Grand Banks data set we determine that the optimum time interval is 5.5 hours (see Appendix C). Using an interval of up to twice this does not significantly degrade predictions; however, the prediction confidence interval can increase drastically for smaller values.

For our operational model we choose the second, computationally simpler method of increasing prediction confidence, given finite noise in the position observation. In calculating the iceberg velocity for a single predictor, we use the last position fix and the earliest fix with an interval not exceeding 6 hours (i.e., if we have fixes at 11:00, 12:37, 14:00, and 16:15, we would use only those at 11:00 and 16:15). The predictor error and confidence intervals are then calculated using the appropriate value of Δt. It is quite probable that the positional error ϵ can be reduced by careful measurement if the iceberg is perceived as a real threat. The model therefore allows the user to change this parameter. The software is delivered with the positional error value of ϵ = 750 m corresponding to the 1984 and 1985 data sets, and caution should be used in changing this value.

3.3 PROBABILITY CALCULATIONS

Garrett et al. (1985) suggest a simple method of evaluating the probability of an iceberg approaching the rig within a certain radius. By taking the radius (R) as equal to the rig size, this calculation may provide a way of comparing predicted impact probability to allowable risk level.

The approach is based on the error calculation for predictions (equation C.5). This error gives the standard deviation of actual iceberg position from the most likely trajectory. The error is assumed to have a normal distribution and is used to compute confidence intervals for the predictions. It can also be used to assess the probability density of iceberg occurrence as:

$$p' = \frac{E^{-2}}{\pi} e^{-s^2/E^2} \qquad (3.3)$$

where s is the distance from the most likely predicted iceberg position. The quantity p' expresses the probability of finding an iceberg per unit area. Integrating this quantity over a circle of radius R gives us the chance of finding the iceberg within the critical circle. Assuming that p' is uniform over the circle, we can calculate the likely fraction of time an iceberg would be within the critical radius as:

$$T_c = \int_0^T \pi R^2 \, p' \, dt \qquad (3.4)$$

where T is the forecast period. Denoting the average time required for an iceberg to cross the critical circle by T_p, we can calculate the number of times the iceberg is likely to cross the critical circle (this is equivalent to the probability of iceberg impact on the rig) as T_c/T_p. Assuming that the iceberg velocity as it crosses the critical circle is normally distributed with zero mean and variance c, Garrett et al. (1985) estimate T_c as $\frac{\pi^{3/2}R}{2\,c}$ yielding a probability of impact of:

$$P = 2\pi^{-1.5}E^{-2}R\ c\int_0^T e^{-s^2/E^2} \tag{3.5}$$

Implicit assumptions in this formula are that:

i) p' can be considered uniform over the critical circle,

ii) iceberg trajectories are relatively straight as they cross the critical circle, and

iii) the iceberg has no net expected velocity as it approaches the rig.

Assumption i) implies $\frac{R\ s}{E^2} \ll 1$. Using equation C.2, from Appendix C, for the prediction error, E, with representative parameters for the Grand Banks, we find that E reaches 5000 m within a few hours of predictions. With an effective rig radius of 300 m we therefore have $\frac{R}{E} < 1$. It can also be shown that for practical purposes $s \lesssim E$ for icebergs close enough to pose a significant threat to the rig. For the Grand Banks operation, assumption i) is therefore appropriate.

Using again a rig radius of 300 m with a typical iceberg speed of 0.25 m/s for the Grand Banks, we find that an iceberg would cross the critical circle in an hour. As seen in Chapter 2 this time is short compared to the time scales of motion on the Grand Banks, and, for all practical purposes, the path of icebergs can be considered straight as they traverse the critical circle.

On the Grand Banks it has been shown that tides, winds, and mean flows contribute significantly to iceberg motion, therefore we cannot assume that the iceberg has no expected velocity as it traverses the critical circle. The velocity of the iceberg as it approaches the rig can be given by:

$$U = \hat{U} + c\,\epsilon \qquad (3.6)$$

where $\hat{U}$ is the expected velocity, and

$c\,\epsilon$ is a random velocity with standard deviation c.

The expected velocity, $\hat{U}$, is given by the predicted iceberg velocity plus a component related to the iceberg position relative to the rig. The latter is required to account for the fact that if the iceberg enters the critical circle it is likely to have an additional component of velocity toward the rig (i.e., with the predicted position at 5000 m from the rig after 10 hours, if the iceberg were to hit the rig after these 10 hours an unpredicted velocity component of approximately 500 m/hr toward the rig could be implied).

Taking $\hat{U}$ into account the average time to cross the critical circle is given by:

$$T_p = \int_0^{\infty} 0.5\pi \frac{R}{V} P(V)\, dV$$

where P(V) is the iceberg speed probability distribution given by:

$$P(V) = \int_0^{2\pi} \frac{V}{\pi c^2} e^{-((\hat{U}-V\cos\theta)^2 + V^2\sin^2\theta)/c^2}\, d\theta$$

These definite integrals can be solved to yield:

$$T_p = 0.5\,\pi^{1.5}\,R\,c^{-1}\,e^{-.5\hat{U}^2/c^2}\,I_o(.5\hat{U}^2/c^2) \qquad (3.7)$$

where I_o denotes the zeroth order modified Bessel function of the first kind (Abramowitz and Stegun, 1964, p. 374).

In the actual model, the radius R is set equal to the sum of the rig and iceberg radii, and the probability of impact is calculated at each time step, Δt = 1 hour, as:

$$P_{\Delta t} = \Delta t \left[[\,\pi\,R^2\,p'\,] \,/\, [\,0.5\,\pi^{1.5}\,R\,c^{-1}\,e^{-.5\hat{U}^2/c^2}\,I_o(.5\hat{U}^2/c^2)\,] \right] \qquad (3.8)$$

The chance of impact is therefore calculated as a function of time, and $P_{\Delta t}$ is cumulated at each model time step to obtain the total probability of impact.

3.4 RIG-OBSERVED CURRENTS

In view of (i) the noise present in position and hence velocity measurements for an iceberg and (ii) the fact that the Eulerian timescale T_E is greater than the Lagrangian timescale T_L (γ^{-1} in earlier sections), it seems possible that the use of current meter information at the rig or platform itself might significantly improve the predictability of an iceberg trajectory.

Let U_o denote the velocity of the iceberg at t = 0 (after removal of the mean flow, the effect of the wind and the tides), U_r the current at t = 0 at the rig (also minus mean flow, winds, and tides) and U_t the residual velocity of the iceberg at time t (i.e., U_t is what we seek to predict), as shown in Figure 3.2. One could consider a two-term predictor:

$$U_t = \alpha_1 U_o + \alpha_2 U_r \tag{3.9}$$

and solve for the coefficients α_1, α_2 that minimize the mean square error in U_t. This correlation will require knowledge of functions, between the various pairs of velocities, which we might denote symbolically as R_{ot} (which is just the Lagrangian autocorrelation, R_L), R_{or} (which is the Eulerian correlation at zero time lag and spatial separation X_o) and R_{rt}. This last correlation is in principle a hybrid Euler-Lagrange quantity and is not easily determined. We could approximate it by the Eulerian correlation function R_E (X_r, t) for time lag t and spatial separation X_r, where X_r is the distance from the rig to the optimal prediction of the iceberg's position.

A detailed treatment of the problem would require attention to the two-dimensionality of the correlations and would also have to adjust for the mean flow, as discussed later. For the moment it seems clear that using U_r will be beneficial if $R_E(X_r,t) > R_L(t)$. Indeed for small times, t, we can expect $R_E(X_r,t) < R_L(t)$ so that one could consider using a one-term Lagrangian predictor at first, possibly switching to a one-term predictor using the rig-observed current at some later time.

Reasonable models for R_L and R_E are:

$$R_L(t) = Ae^{-t/T_L}, \text{ and } \qquad R_E(X_r, t) = e^{-|X_r|/L_E}\, e^{-t/T_E} \tag{3.10}$$

Where A allows for noise in iceberg velocity estimates, we have assumed that the rig residual current corresponds exactly to the residual current affecting

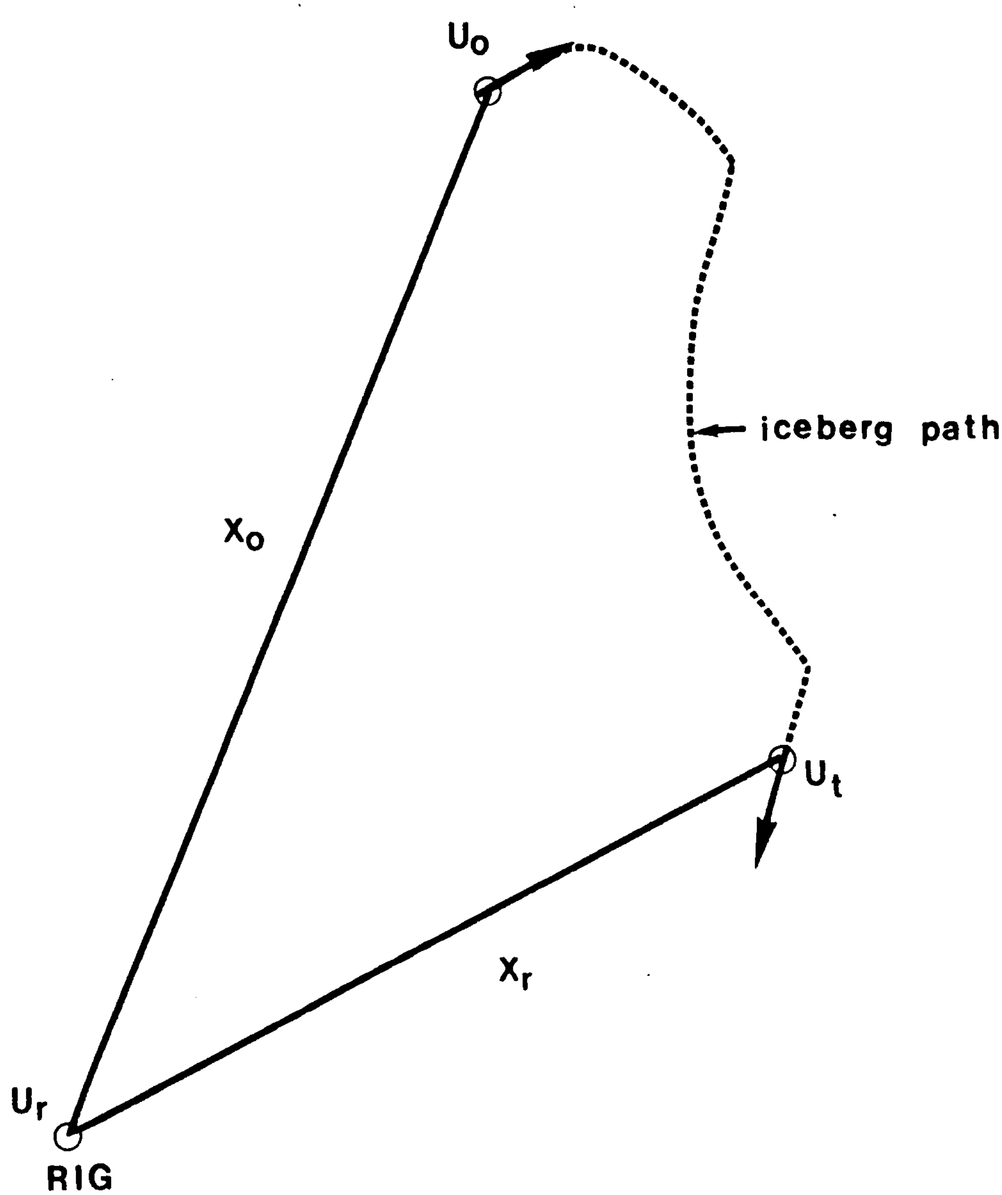


Figure 3.2. Definition sketch for iceberg and rig-observed velocities.

iceberg motion, and we have ignored the distinction and difference between longitudinal and transverse velocity components in the Eulerian correlations.

Rig currents are thus likely to be valuable if:

$$|X_r| < L_E \, (t/T_L - t/T_E - \ln A). \qquad (3.11)$$

where L_E is the Eulerian length scale. For the Grand Banks we may take $A \simeq 0.8$ (assuming the optimum Δt for position differencing, see Appendix C), $T_L \simeq 25$ hours, with rough estimates of $T_E \simeq 40$ hours and $L_E \simeq 30$ km, so that for rig currents to be valuable requires $|X_o| < -30 \ln A = 6.5$ km. It is unlikely that $|X_o|$ will ever be this small. For $t = 24$ hours we would require $|X_r| < 17.5$ km, which makes it appear that rig currents could indeed be valuable for the later stages of prediction. However, it must be remarked that, in the presence of a mean flow $\overline{U}$, the correct separation to use in the Eulerian correlation is not $|X_r|$ but rather $|X_r - \overline{U}\, t|$ which might be somewhat greater than $|X_r|$. After 24 hours no predictor is very successful. The value of rig currents is further reduced if there is vertical shear of the current and one does not know the appropriate depth average to apply to a particular iceberg.

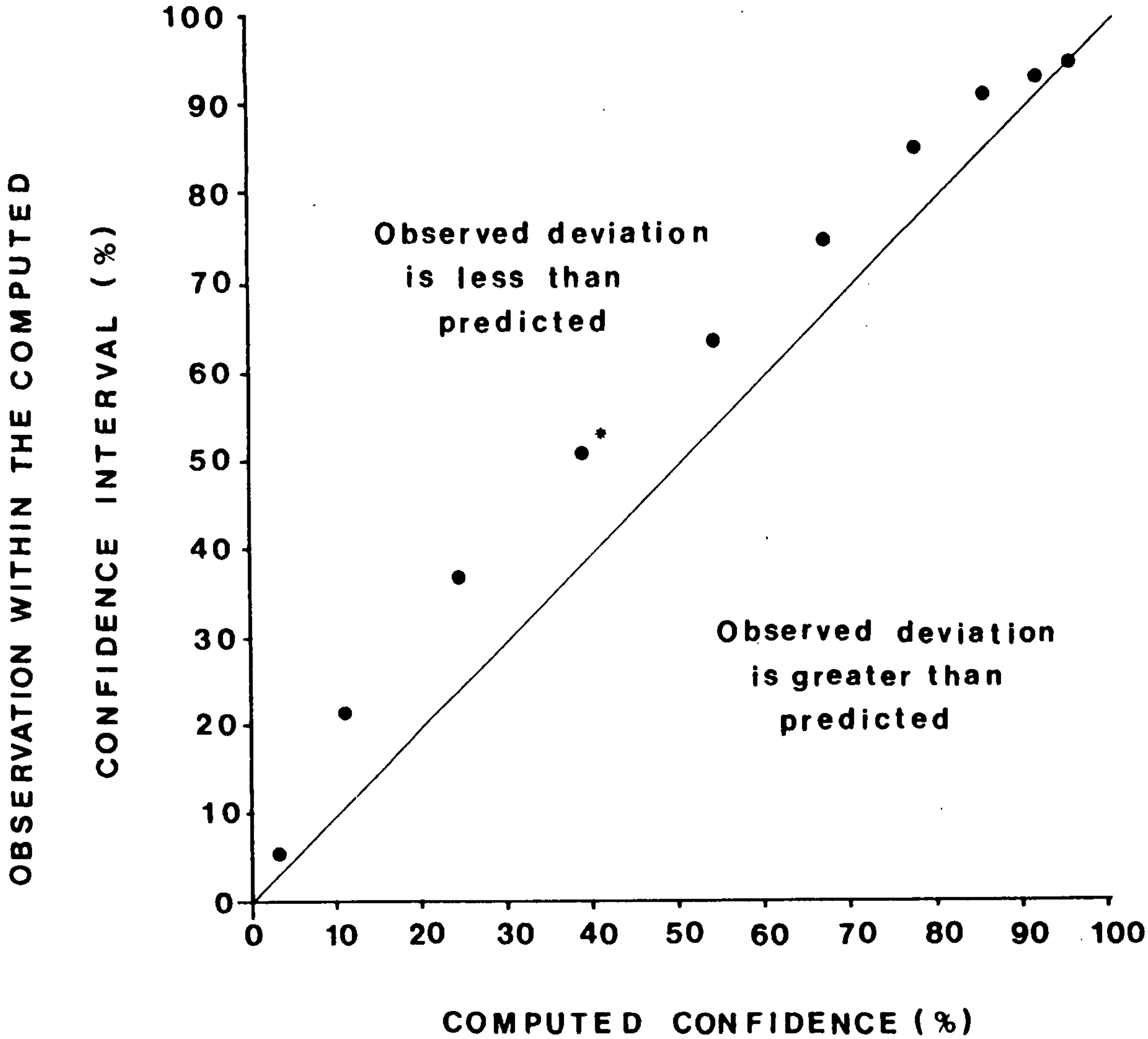
In summary, it does not seem at this stage that the use of rig-observed currents would lead to a significant improvement in trajectory prediction, but the topic has not been studied in detail.

3.5 MODEL VERIFICATION

To verify the correct operation of the model we have compared predicted and observed iceberg trajectories for 1984 and 1985. Figure 3.3 shows a cumulative comparison of predicted confidence intervals with the deviations between observed and predicted positions. As can be seen the observed deviations are less than the computed confidence intervals, indicating that the model performs slightly better than anticipated. Figure 3.4 shows two sample plots of an observed iceberg track, overlaid on the predicted trajectory and confidence interval.

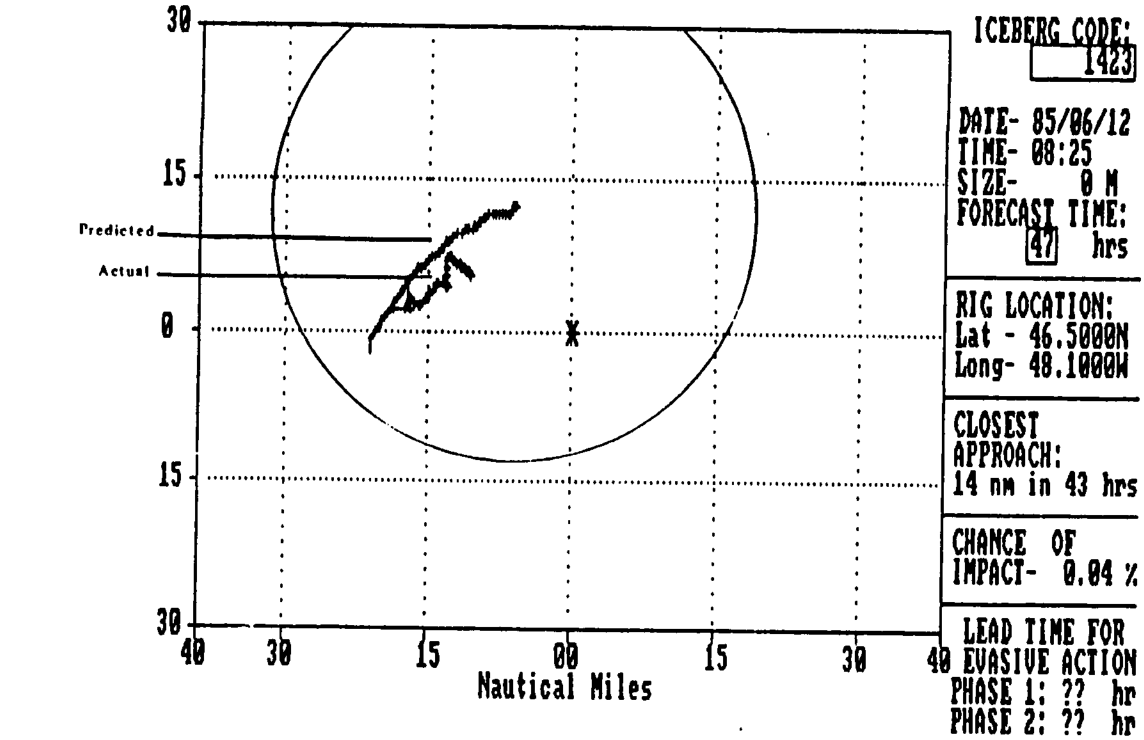
It should be noted that the verification herein does not use an independent set of data (i.e., the same data are used to determine the model parameters and for verification). This verification exercise is therefore aimed more toward verifying the correct implementation of our conceptual iceberg model. By not including an independent data set for verification we implicitly assume that mean iceberg behaviour is well characterized by the available data, and that this behaviour will not change in the future. The fact that the model gives adequate results for both the 1984 and 1985 data, which we have seen in Chapter 2 to be markedly different, lends credibility and confidence to our procedure.

The actual prediction error depends on model input (number of fixes and time intervals), as well as iceberg location. Figure 3.5 is a dimensional plot of error as a function of time, the nondimensional version of which can



* Interpretation: 50% of observations lie within the computed 40% confidence interval, model predictions are therefore slightly better than anticipated.

Figure 3.3. Comparison of observed and computed confidence, based on 1098 observations.



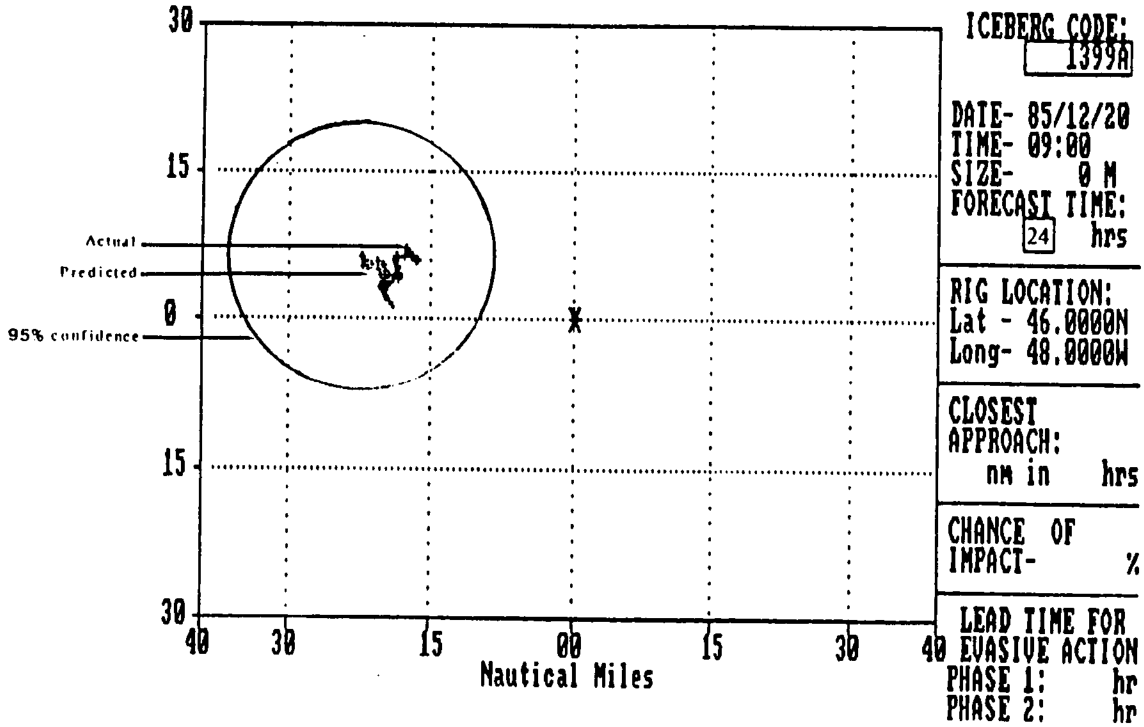


Figure 3.4. Model prediction for two observed tracks.

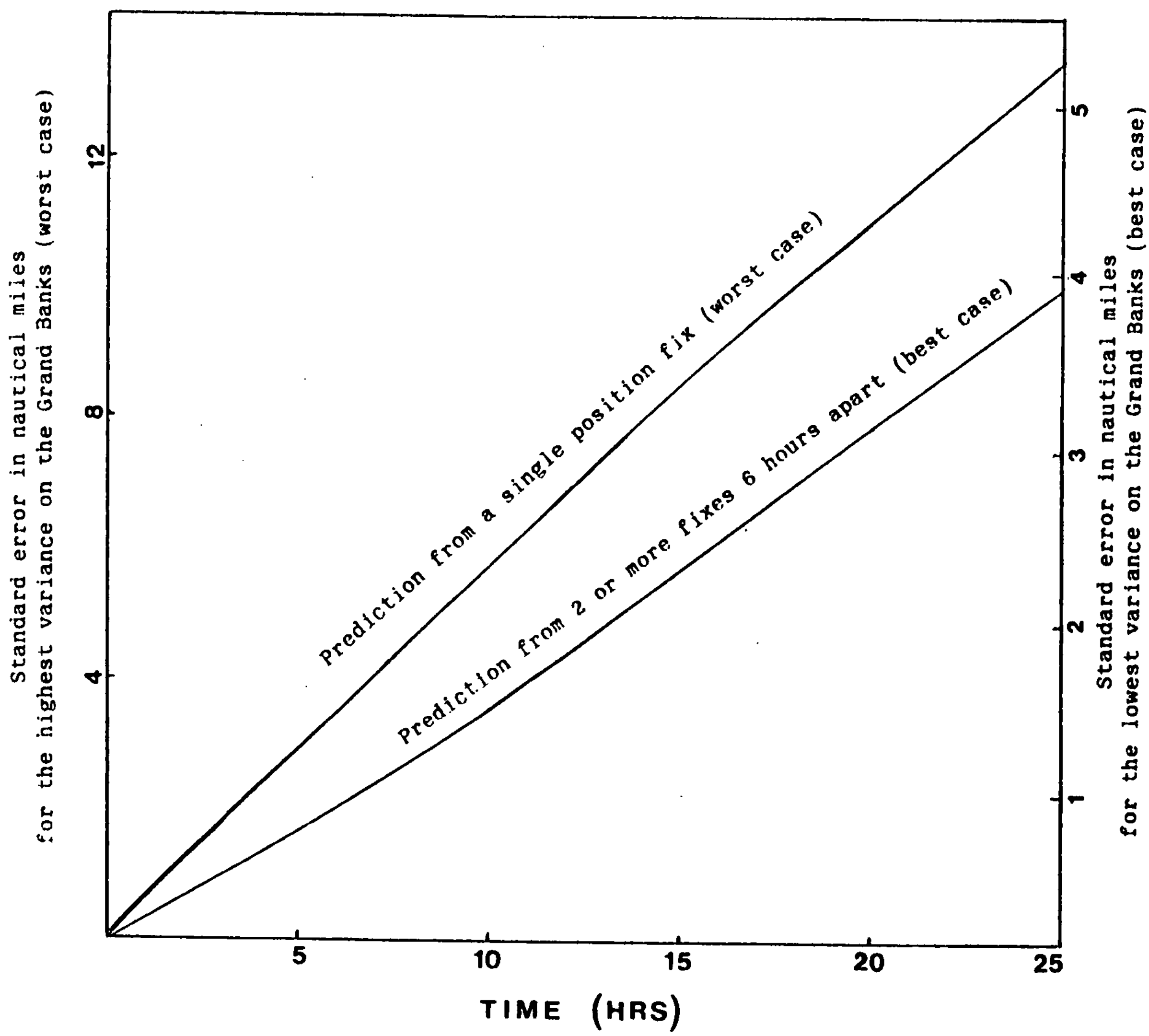


Figure 3.5. Dimensional standard error, E, for the optimum observational time interval.

be found in Appendix C. The error depends on the local velocity variance, which varies spatially, so that two scales are given, corresponding to the lowest and highest variances found in the study area. The two curves correspond to prediction error given a single position fix (worst case), and given two or more fixes over a period of 6 hours (optimum case). As can be seen, for a 10-hour prediction, the standard error can range between 1.5 and 6 nautical miles; after 25 hours the error ranges between 4 and 14 nautical miles (the radius of the confidence circles in Figure 3.4 are 1.65 times the standard error). These errors are admittedly large if we consider that an iceberg would typically move by 10 or 15 nautical miles per day. This error, however, is intrinsically linked to our present inability to forecast oceanic eddy motion, and will persist in any iceberg prediction scheme until major advances are made in current forecasting techniques.

CHAPTER 4 MODEL IMPLEMENTATION

The model presented in Chapter 1 and extended in Chapter 3 has been implemented on an IBM PC microcomputer. The language of implementation is PASCAL, chosen because it offers a superior development tool and results in an efficient machine code for microcomputers.

In its present implementation the model is applicable for icebergs located between $45^{o}50'$ - $47^{o}30'$N and between 47^{o} - 49^{o}W. The only intrinsic assumption in the model is that the behaviour of icebergs on the Grand Banks will not change systematically in the future, which seems a reasonable assumption.

A significant advantage of the present model over others is that it provides estimates of probability of iceberg impact upon a platform. By calculating this probability as a function of time the model also computes an optimum time for evasive action by requiring that the chance of impact never exceeds the allowable risk (set by the operator).

4.1 MICROCOMPUTER FOR IMPLEMENTATION

The model is implemented on an IBM PC (or compatible) computer. These computers are widely available and easily serviced. Rugged portable units can be purchased at a competitive price compared to a simple graphic terminal which would be required for implementation on the more common shipboard mini-computers.

PASCAL was used as the computer language of implementation principally because it is in many respects a superior development tool for micros and yet retains a significant level of universality. The model makes extensive use of graphic display, pop-up menus, and special function key entries, all of which contribute to an ease of operation and of interpretation. Although PASCAL compilers are available for most computers (mainframe, mini, or micro) hardware-specific features of the IBM PC make it necessary to use non-standard language elements. The model is, therefore, not easily portable to other systems. The numerical algorithm section of the model is coded in standard PASCAL and could be ported to other installations; however, it represents a relatively small portion of the code and of effort involved in development. The model is delivered in compiled version (executable machine code) and can complete most predictions within a minute.

4.2 MODEL ASSUMPTIONS AND APPLICABILITY

The only implicit assumption in the model is that the behaviour of icebergs will not change systematically in the future. This assumption seems reasonable, because the model has been successfully verified against 1984 and 1985 data, which show marked differences. We do not expect interannual variability to cause serious problems.

Several other assumptions concerning second-order effects of spatial variability and observational noise have been made, and shown to be adequate for the Grand Banks (see Chapter 3 and Appendices A and B), and therefore should not be of immediate concern to the model user. The principal limitation is the restriction to icebergs between $45^{o}50'$ - $47^{o}30'$N and between 47^{o} - 49^{o}W. Although the model will provide predictions for icebergs outside this region (using tidal constants, mean flow, and variance defined for the closest grid cell) the results may not be very accurate. Another limitation involves the extent of the forecasting period. Although the model will provide forecasts for any length of time, the wind forecasts used in the prediction only extend 48 hours into the future. Predictions for times greater than this use the wind forecast at 48 hours, and will decrease in accuracy as this wind speed becomes less applicable.

It should be kept in mind while using the model that prediction accuracy is increased as more position fixes become available for a particular iceberg, for up to 6 hours (i.e., the model uses only fixes within the last 6 hours). Because of this and also because the iceberg drift will actually change with each observation, the model predictions will change as new fixes are entered. Therefore, it is recommended that a tight monitoring schedule be maintained for any iceberg posing a significant threat.

4.3 DETERMINATION OF OPTIMUM TIME FOR EVASIVE ACTION

The model computes the cumulative probability of iceberg impact upon the rig. This probability may in itself provide valuable information by giving a measure of the threat posed by a given iceberg, but it lacks an element of timing which is required for decision making. To provide this timing information the model computes the probability of impact as a function of time. This function can be related to the allowable risk and the necessary lead time for evasive action as shown in Figure 4.1. In this diagram important parameters are:

- $T_{decision}$, the amount of time available before undertaking evasive action,
- P_{total}, the total cumulative probability of iceberg impact,
- $P_{allowable}$, the allowable risk level,
- $T_{allowable}$, the time at which P_{total} exceeds $P_{allowable}$,
- T_{max}, the most likely time of impact, and
- T_{lead}, the lead time required to complete evasive action.

These parameters, except for T_{lead} and $P_{allowable}$, depend on the most recent iceberg observations and may change as new position fixes become available.

In cases were $P_{total} < P_{allowable}$, no evasive action is recommended and $T_{allowable}$ as well as $T_{decision}$ are undefined. Otherwise $T_{decision}$ is given by:

$$T_{decision} = (T_{lead} - T_{allowable}), \text{ or}$$
$$(T_{lead} - T_{max}), \text{ whichever is smallest.}$$

This definition will ensure that the allowable risk level is never exceeded and that evasive action is taken early enough to be effective at the most probable time of impact.

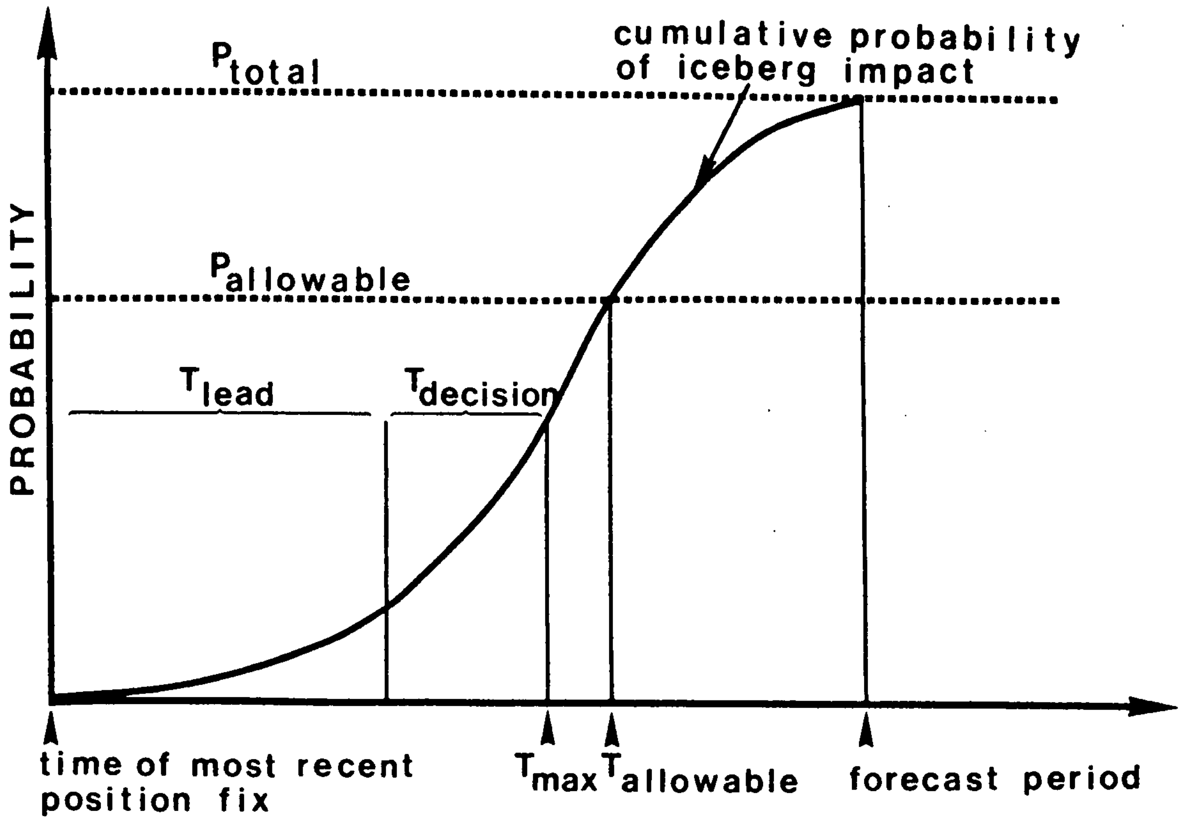


Figure 4.1. Probability of iceberg impact as a function of time, and its relationship to allowable risk and required lead time for evasive action.

Several lead times can be used in the calculation, corresponding perhaps to various stages of evasive procedures. For example, 24 hours of lead time may be required to pull up the pipe etc. whereas only 4 hours may be required to drop the last anchors and move. The model allows setting two independent lead times labelled as Phase 1 and Phase 2 but uses a common allowable risk level. The model computes the two values of $T_{decision}$, which are labelled Lead Times for Phase 1 and Phase 2 in the model output. These are in fact estimates on how much lead time is left before a decision must be made. Evasive action is probably best delayed until this time because new position fixes obtained in the interim will improve the prediction and may reduce the probability of impact.

The calculation of allowable risk is an actuarial problem best undertaken by the operators, because they are more knowledgeable of the implications of rig loss. We would like to point out that although such calculations may seem cynical, allowable risk is always considered either explicitly or implicitly. The use of safety factors and/or worst case scenarios is equivalent to selecting a low allowable risk, which in iceberg management is rarely explicitly calculated, even though it is common practice in aspects of operational or design engineering. From an actuarial point of view, it is much preferred to select the allowable risk level explicitly, based on cost analysis or other specific guidelines. As an example, an allowable risk level of 1.0% is adequate for a ratio of rig disconnect to rig loss costs of 1/100 (i.e., the typical rig disconnect cost may be $1 000 000 for 5 days of down time for evasive and reentry procedures, whereas the cost of rig loss would be \$100 000 000). An alternate method of computing allowable risk could consider that the probability of rig loss is not to exceed one chance in a hundred years (rp = 100). If the mean number of icebergs threatening operations on the Grand Banks, n, is 100 per year, then the allowable risk for an individual iceberg would be $1 - (1 - \frac{1}{rp})^{1/n}$, which for $rp >> 1$ is approximately $\frac{1}{n\ rp}$ or 0.01%, given the present parameters.

CONCLUSIONS

The conceptual model for iceberg prediction, proposed by Garrett et al. (1985), has been implemented for the Grand Banks. Although several enhancements have been necessary for its operational implementation, the essence of the model remains the same. The usefulness of such a forecasting tool depends not only on its theoretical foundation but also on practical implementation features ensuring ease of use and clarity of the forecast products. To this end the design process has included the production of several prototypes that have been reviewed by representatives of the oil industry and government with an interest in improving the capabilities of forecasting iceberg trajectories. The model shows promise as a useful operational tool; however, this promise will only be confirmed after verification and field implementation of the model.

This analysis, specifically to determine the drift characteristic of icebergs, is of considerable interest not only for oceanographic purposes, but also for other practical applications. Because icebergs are natural drifters, which, at least in some years, occur in great numbers, our analyses were able to confirm results of earlier studies on the circulation patterns on the Grand Banks, and to provide greater spatial resolution and accuracy. The techniques and results obtained here are applicable in forecasting the drift of any material or object on the water surface. Thus, the present model could provide an excellent starting point for the development of a realistic forecasting system for oil spill trajectories. Other applications could include search-and-rescue operations in which optimization of search patterns requires consideration of the probability density as well as of the predicted most-likely position.

RECOMMENDATIONS

To make the most of the analysis and development undertaken in this study, several recommendations are in order. Restricting ourselves only to the problem of iceberg forecasting, we have ranked four topics for further investigation:

i) field implementation of the forecasting model,

ii) further investigation of eddy variability on the Grand Banks,

iii) the use of an array of telemetering current meters to improve forecastability, and

iv) the development of an eddy resolving model of local ocean circulation.

A monitored field introduction of the model is essential. Ensuring the acceptance of this new forecasting tool on the oil rigs involves two aspects of interaction with onshore and offshore personnel. First, seminars and short training courses may be appropriate to introduce the new techniques to the operators and to their environmental support groups. These seminars should be geared to a proper understanding of the model operation and predictions. The second, and probably the most important aspect, is the compilation and incorporation of comments and suggestions from the actual users of the model. Although many efforts have been made to ensure practical implementation of the model, field trials are likely to bring out unanticipated shortcomings. Many model enhancements could be considered now (such as the simultaneous consideration of many icebergs, and the prediction of the most effective evasive route) however, to ensure that these are of practical value they would be best based on user feedback. To have a fully developed model within a reasonable amount of time it is recommended that field trials and monitoring be undertaken during the 1987 iceberg season so that an upgraded model would be available for the following year.

Although our analysis has improved our understanding and knowledge of iceberg drift on the Grand Banks, several questions remain unanswered. For example, the diffences noted between the 1984 and 1985 data have been postulated as reflecting spatial variability (Section 2.5), but have not been considered in detail. An analysis of iceberg velocity autocorrelation and wind/iceberg cross-correlation, including consideration of spatial variability, would be particularly appropriate, especially as more data become available after the 1986 season. If the features detected in the 1985 data are confirmed as originating from diurnal shelf waves, their inclusion in the model could also significantly improve predictions, especially near the shelf edge.

The usefulness of using rig-observed currents in the prediction scheme was considered in Section 3.4. It was concluded that, from a practical point of view, moderate improvements in trajectory forecasts could be obtained by using rig-observed currents. Although their use may be a worthwhile improvement to the present forecasting model, the improvement becomes significant only when the iceberg gets close to the rig, which is a situation that we are trying to avoid. More useful predictors would be currents monitored from an array of telemetering instruments moored around the rig. Such current meters would reduce the effect of observational error in iceberg fixes, thus directly improving forecast accuracy. The major advantage may, however, be due to the fact that Eulerian time scales are generally greater than their Lagrangian counterparts, implying that eddy motion could be effectively predicted for periods greater than 25 hours. A preliminary feasibility study based on presently available data could determine the optimum current meter spacing, as well as provide better estimates of forecast improvement for a cost benefit analysis.

We have taken a statistical approach to predict the motion resulting from oceanic eddies, however it is theoretically possible to predict these deterministically (Robinson et al., 1984). Meteorologists, for example, are using deterministic techniques to forecast storms which are the atmospheric equivalent of ocean eddies. The principal difference between the two applications is the spatial scale involved for the atmospheric and oceanic phenomena. A model for resolving local eddies for the ocean would require a rather small grid size, yet must consider a portion of the ocean large enough so that eddies entering the boundaries cannot propagate to the rig within the forecast period. To initialize the model and provide continuously changing boundary conditions an array of instruments is required to monitor ocean movement continuously within the model domain. Such a forecasting system would bear a cost comparable to weather forecasting, which is large in relation to current expenditures for iceberg trajectory prediction. Such a system may, however, be justified in view of the cost of disconnecting a rig. The future of iceberg trajectory forecasting depends largely on the introduction of practical modelling tools. With this study, we have taken a large step in this direction. Although the confidence intervals of the present model are admittedly large, the model should perform within the stated accuracy. This level of error is common to all present iceberg models and is the most precision practically possible at this time. Whether or not further work is undertaken depends on the acceptance of the present model. It is therefore essential that this tool be introduced in the field promptly with proper training and monitoring.

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APPENDIX A

EFFECTS OF SPATIAL VARIABILITY OF VELOCITY STATISTICS

EFFECTS OF SPATIAL VARIABILITY OF VELOCITY STATISTICS

Riley and Corrsin (1974) have pointed out that local mean shear will lead to an accelerated growth in rms particle dispersion over that predicted by simple Taylor diffusion. To determine if such accelerated growth is important for the Hibernia region we shall consider the simple situation of a linear velocity gradient defined by:

$$\overline{v} = v_o + s\,x \quad \text{and} \quad \overline{u} = 0$$

where v_o and s are constants and with an isotropic variance of V^2. Riley and Corrsin (1974) have determined the mean squared x and y components of particle position for such a velocity field as:

$$\overline{x'^2} = 2\,V^2 \int_0^t\int_0^\tau R(\beta)\,d\beta\,d\tau \qquad (A.1)$$

$$\overline{y'^2} = s^2V^2 \left[\frac{2}{3} t^3 \int_0^t R(\tau)\,d\tau - t^2 \int_0^t \tau\,R(\tau)\,d\tau + \frac{1}{3}\int_0^t \tau^3 R(\tau)\,d\tau \right] + \overline{x'^2} \qquad (A.2)$$

The increase in $\overline{v}$ with x results in an accelerated diffusion in the y direction with time. Initially this effect is small as the eddy variability V is much greater than that due to shear. However, at sufficiently large times, the shear term in equation A.2 will dominate and

$$\overline{y'^2} \simeq \frac{2}{3}\,s^2\,V^2\,t^2\,\gamma^{-1} \qquad (A.3)$$

To determine the importance of shear diffusion for iceberg modelling on the Grand Banks we have computed $\overline{x'^2}$ and $\overline{y'^2}$ using an exponential autocorrelation function with a time scale of 25 hours and a value of s = (0.07 m/s) / (10 km). The results, normalized by γ / V, are presented in Figure A.1. These show that after two integral time scales, (50 hours), the shear results in only a 20% increase in rms iceberg displacement. Since operational iceberg forecasting is concerned mostly with shorter times, and considering the other uncertainties in our statistics, it is reasonable to ignore the influence of shear on the rms iceberg positions. Shear in mean velocity will be included only insofar as it affects the predicted most likely trajectory and not in the calculation of confidence intervals.

Freeland et al. (1975) have shown that spatial dependence of diffusion coefficient will result in a mean up-gradient Lagrangian drift even though the mean eddy velocity is assumed identically zero. This is analogous to turbulent entrainment processes where fluid at rest is being entrained into an adjacent turbulent region. The diffusion coefficient is related to the eddy energy V^2 through $K = V^2 \gamma^{-1}$, and the Freeland et al. (1975) relation for mean Lagrangian transport can be written as:

$$\overline{u} = \gamma^{-1} \nabla V^2 \qquad (A.4)$$

From our observations we can estimate the magnitude of ∇V^2 as $(0.05\ m^2/s^2)$ / (100 km) yielding with γ^{-1} = 25 hours, a drift of 0.04 m/s. This drift is small compared to the scale of mean currents in the study area and in any case is included in our analysis because the mean flow patterns are derived from Lagrangian velocity estimates.

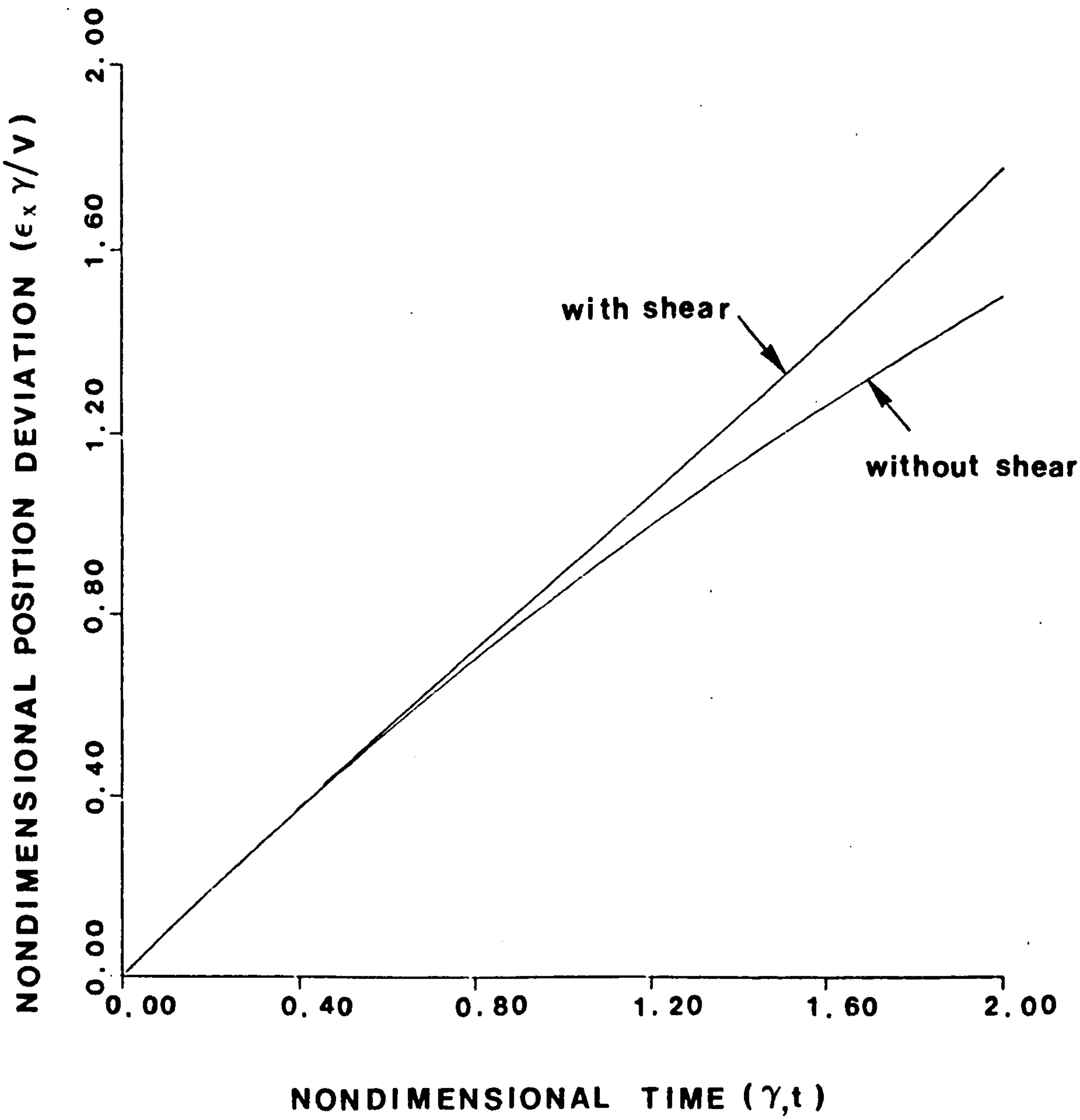

Figure A.1. Normalized rms deviation of position, with and without the consideration of mean shear.

APPENDIX B

A MULTI-TERM PREDICTOR FOR THE GRAND BANKS

A MULTI-TERM PREDICTOR FOR THE GRAND BANKS

In the case where we have a set of velocity estimates obtained from successive position observations, taken Δt apart, and each with an uncorrelated error, ϵ, we have in one dimension:

$$u_{o\,n} = (x_{n+1} - x_n + \epsilon_{n+1} + \epsilon_n) / \Delta t \qquad (B.1)$$

The observed velocity variance is then:

$$\overline{u_o^2} = \overline{u^2} + 2\,\overline{\epsilon^2} / \Delta t^2 \qquad (B.2)$$

where $\overline{u_o^2}$ is the observed variance,

$\overline{u^2}$ is the true variance, and

$\overline{\epsilon^2}$ is the variance of positional errors.

Substituting $2\,\overline{\epsilon^2}/\Delta t^2$ for $\overline{\epsilon_u^2}$ in equation 1.8 gives $A = \left[1 + \frac{2\overline{\epsilon^2}}{\overline{u^2}\,\Delta t^2} \right]^{-1}$. The observed discrete autocorrelation is given by:

$$R_o(\Delta t) = A\,R(\Delta t) - \frac{1}{2}(1 - A), \text{ and}$$
$$R_o(n\,\Delta t) = A\,R(n\,\Delta t),\ n>1. \qquad (B.3)$$

Introducing this into equation 1.4 with $R(\tau) = e^{-\gamma t}$ and assuming that the observational interval, Δt, is small compared to γ^{-1}, we can obtain the following expression for the coefficients α_n of a N term predictor:

$$\alpha_n = A\,\gamma^{-1}\,e^{-\gamma\Delta t/2}\,(1 - e^{-\gamma t})\,\beta_n, \qquad (B.4)$$

with β defined by:

$$\begin{bmatrix} 1 & Ae^{-\gamma\Delta t}-.5(1-A) & Ae^{-2\gamma\Delta t} & \cdot & \cdot \\ Ae^{-\gamma\Delta t}-.5(1-A) & 1 & Ae^{-\gamma\Delta t}-.5(1-A) & \cdot & \cdot \\ Ae^{-2\gamma\Delta t} & \cdot\ \cdot\ \cdot & 1 & \cdot & \cdot \\ \vdots & & & & \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \end{bmatrix} = \begin{bmatrix} 1 \\ e^{-\gamma\Delta t} \\ e^{-\gamma 2\Delta t} \\ \vdots \end{bmatrix}$$

For $\gamma\, \Delta t$ = 12.5 and A = .65, which is representative for the Grand Banks iceberg data set, we have β = 1 for N =1 and β = (0.74, 0.63) for N = 2. The prediction error (normalized by $(\gamma^2 / \overline{u^2})^{.5}$) is given by:

$$E(t) = \left[2(\gamma t - 1 - e^{-\gamma t}) - A(1 - e^{-\gamma t})^2 e^{-\gamma \Delta t/2} \sum_1^N \beta_n e^{-\gamma (n-1)\Delta t} \right]^{.5} \qquad (B.5)$$

The first term in the bracket is the error variance using no predictor at all, and results from modelling the drift as a simple Taylor diffusion. The second term is the decrease in variance from the use of one or more predictors, which will be denoted by PV for the predictor variance. Figure B.1 shows the function E(t) for various number of predictors, N, with A = 0.65 and Δt = 2 hours which are representative of the Grand Banks data set. Also shown is E(t) in the case A = 1, which represents the best possible prediction given absolute accuracy in position observations. As may be seen the increase in predicted variance for the two-term predictor is about 37% as compared to the one-term predictor (i.e., $PV_{N=2} / PV_{N=1} = 1.37$). This reduction, however, remains a small proportion of the total variance since PV / E^2 is typically 0.35. The net gain in accuracy from a two-term predictor instead of a one-term predictor is about 10% in terms of total variance or 5% in terms of standard error.

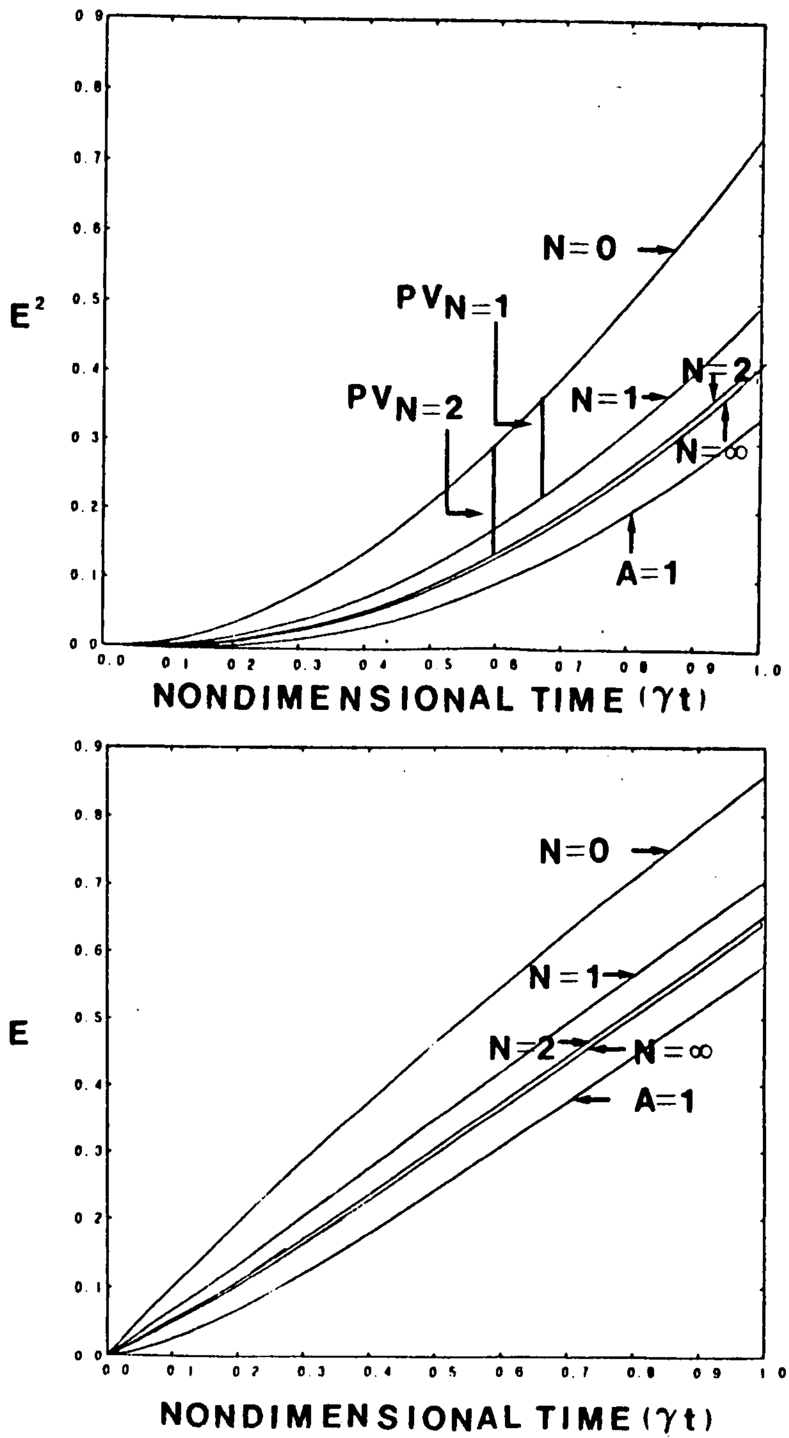

Figure B.1. Normalized standard error, E, and variance, E^2, for various numbers of predictors, with and without observational noise (Δt = 2 hours, γ^{-1} = 25 hours, and A = 0.65).

APPENDIX C

OPTIMUM TIME INTERVAL FOR A SINGLE PREDICTOR

OPTIMUM TIME INTERVAL FOR A SINGLE PREDICTOR

To decrease the effects of noise, instead of using two predictors we can investigate the use of a single predictor using a time interval of $2\Delta t$ between observations ($u_n = (x_{n+2} - x_n + \epsilon_{n+2} + \epsilon_n) / 2\Delta t$). The factor A then increases from 0.65 to 0.88. The reduction in variance, PV, is 28% greater than for the N = 1 case discussed in Appendix B with A = 0.65 and an interval of 1 hour. We have, therefore, made almost the same gain in accuracy as using a two-term predictor, using a computationally much simpler method. There must obviously be a limit to which we can effectively increase the observational interval since if we make it very large the resulting velocity estimate will bear little correlation to the present or future iceberg velocity. The following addresses this question and determines the optimum observational interval which is a function of positional error, ϵ, as well as Lagrangian time scale, γ^{-1}. We also derive a more appropriate formula for prediction error when the observational interval is a significant fraction of the integral time scale.

We want to predict the change in position x in terms of input p. The prediction formula is: $\hat{x} = \alpha p$ with $\alpha = \overline{xp} / \overline{p^2}$. The mean square error is $\overline{x^2} - (\overline{xp})^2 / \overline{p^2}$, so for optimum prediction we want to maximize $(\overline{xp})^2 / \overline{p^2}$.

Here we have $x = \int_0^t u(t')\,dt'$, with u(t) the current speed, and the predictor is the change in position from -T to 0, is given by:

$$p = \int_{-T}^{0} u(t')\,dt' + \epsilon_0 - \epsilon_T,$$

where ϵ_o and ϵ_T are the positional errors at times -T and 0, assumed to be independent with zero mean and equal variance. Using these equations for x and p with a velocity autocorrelation $R(\tau) = e^{-\gamma\tau}$ we can derive:

$$\overline{xp} = \overline{u^2}\int_o^t dt' \int_{-T}^o dt'' \, R(t'-t'') = \overline{u^2}\gamma^{-2} \, (1-e^{-\gamma t}) \, (1-e^{-\gamma T})$$

$$\overline{p^2} = \overline{u^2}\int_o^T dt' \int_o^T dt'' \, R(t'-t'') + 2\overline{\epsilon^2}$$

$$= 2\overline{u^2}\int_o^T dt' \int_o^{t'} dt'' \, e^{-\tau(t'-t'')} + 2\epsilon^2$$

$$= 2\overline{u^2} \, \gamma^{-2}(\gamma T-1+e^{-\gamma T}) + 2\overline{\epsilon^2}$$

from which we can form the ratio $\overline{xp}^2 / \overline{p^2}$. We therefore wish to choose T, the time interval for the predictor, to maximize the following function:

$$F(\gamma T) = \frac{(1-e^{-\gamma T})^2}{\gamma T-1+e^{-\gamma T}+a}$$

where $a = \dfrac{\gamma^2\overline{\epsilon^2}}{\overline{u^2}}$. Requiring that $\dfrac{dF}{dT} = 0$ we get:

$$[(\xi-1+e^{-\xi})+a] \; 2e^{-\xi}(1-e^{-\xi}) - (1-e^{-\xi})^2(1-e^{-\xi}) = 0$$

$$\text{or} \quad \xi+a = \sin \xi \approx \xi+\frac{1}{6}\xi^3 + \ldots$$

so that $T = \left[\dfrac{6\gamma^{-1}\overline{\epsilon^2}}{\overline{u^2}}\right]^{\frac{1}{3}}$

For the Grand Banks, $\overline{\epsilon^2}$ is $(750m)^2$, γ^{-1} is 25 hours and $\overline{u^2}$ is typically $(0.2 \text{ m/s})^2$, so that prediction error is minimized for T = 5.5 hours.

It is interesting to note that as shown in Figure C.1 the function $F(\gamma T)$ changes only slowly for T greater than the optimum value. Therefore choosing T up to 10 hours would not significantly affect the prediction. Using a

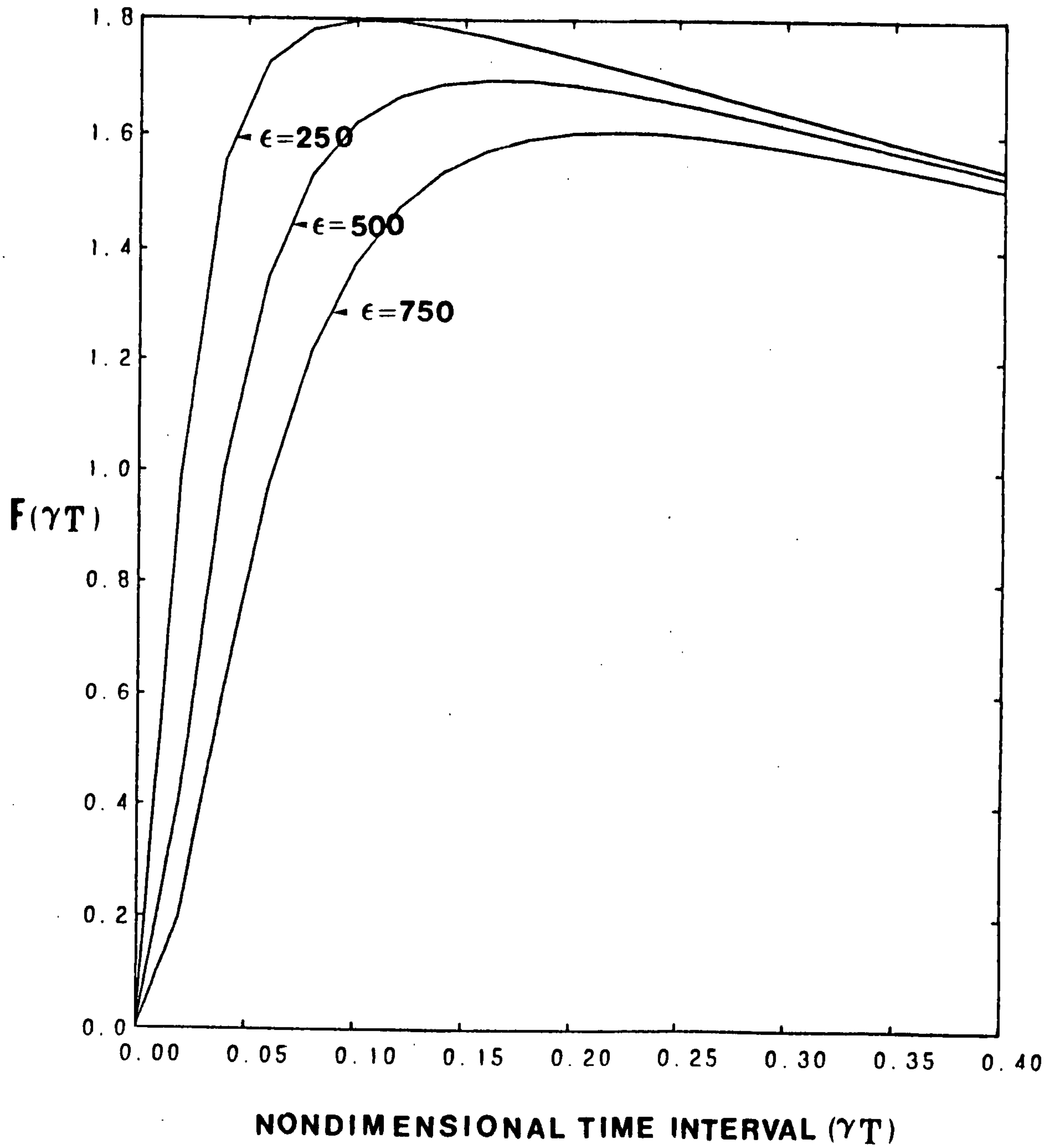

Figure C.1. Function $F(\gamma t)$ describing the relative increase in prediction accuracy as a function of observational interval, T, for γ^{-1} = 25 hours, $\bar{u}^2 = (0.2\ m/s)^2$ and with $\bar{\epsilon}^2 = (750\ m)^2$, $(500\ m)^2$ and $(250\ m)^2$.

smaller time has a much greater effect. It is also worth noting that the maximum value of F does not depend very strongly on the position error ϵ, implying that as long as the positional time interval is equal or greater than the optimum value the effect of positional error is small.

The normalized prediction error can be given by:

$$E^2 = 2(\gamma t-1+e^{-\gamma t}) - \frac{(1-e^{-\gamma t})^2 (1-e^{-\gamma T})^2}{2(\gamma T-1+e^{-\gamma T}) + 2(\gamma^2\epsilon / \overline{u^2})}$$

which is compared in Figure C.2 with the multipredictor error. As can be seen, using an optimum time interval offers practically the same advantage.

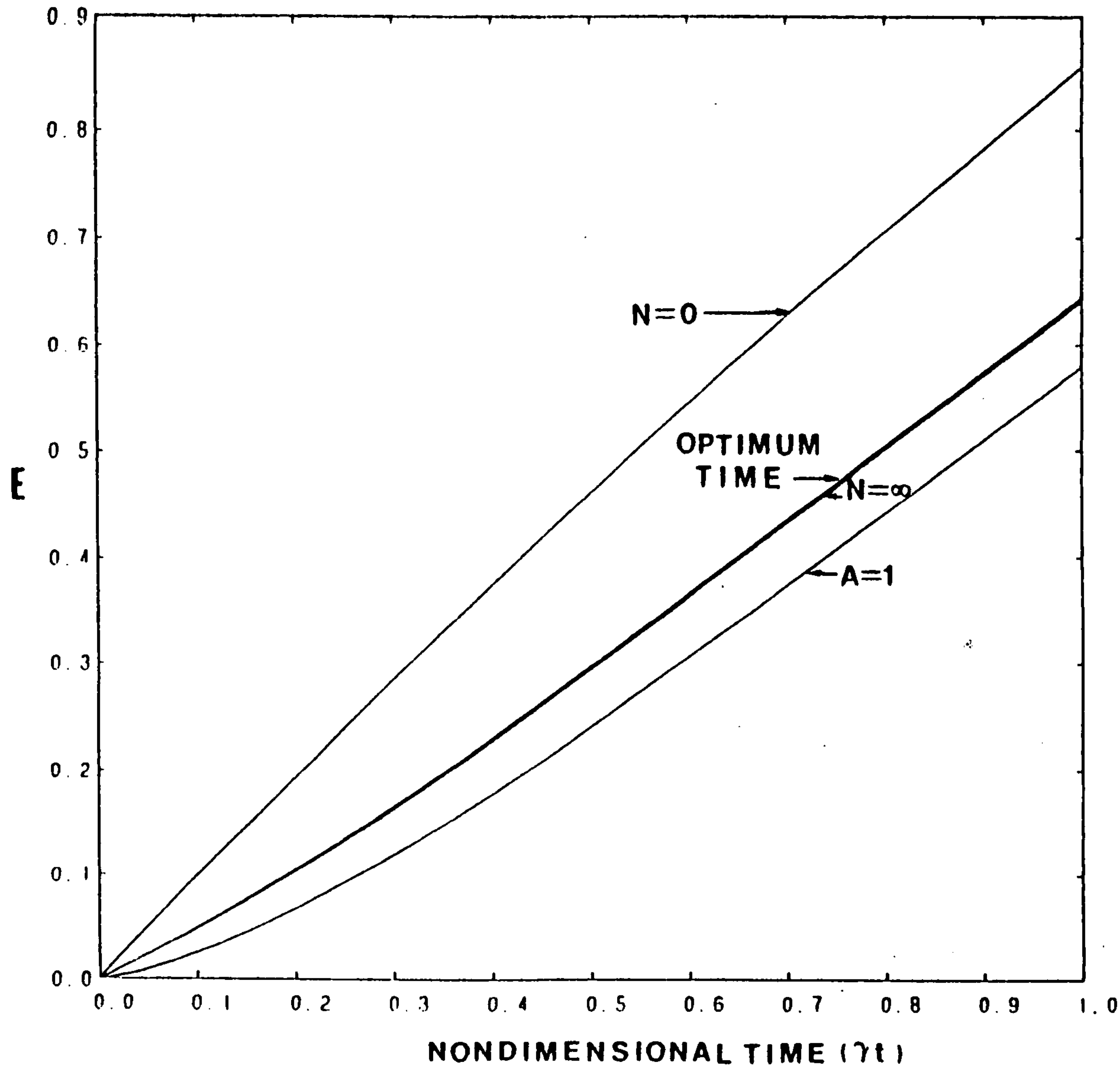

Figure C.2. Nondimensional standard error, E, for the optimum observational time interval.